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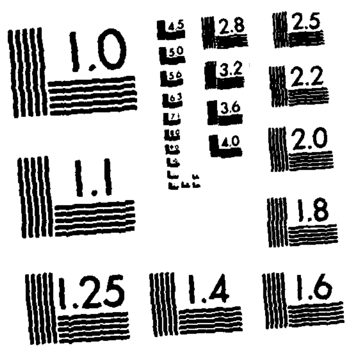

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AN EXAMINATION OF THE

EFFECTS OF COMMAND AND CONTROL

ON THE FORWARD AIR DEFENSE

THESIS

AFIT/GOR/MA/82-D5

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THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

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by

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Preface

The complete development of a model is a difficult but worthwhile experience. It requires an understanding of the problem and the need for the model; a familiarity with similar models and the techniques available. The actual development involves decisions on what parameters to include and how to capture aspects which aren't clearly defined.

In choosing this type of thesis, I was forced to deal with these types of problems. In essence, it is an attempt to learn how to think through a problem and quantify the unquantifiable. My personal return from this effort and the development of my skills as an analyst is many times the contribution of this study.

I would like to thank my advisor, Lt. Col. James Bexfield for his guidance; Major Dave Maghee, Major Jack Bogusch, and Captain Chuck Williams for sharing their real world knowledge with me. Above all others, I thank my husband, Mike, for his love, patience, and understanding through the many sleepless nights and long days apart.

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Abstract

This study develops a basic methodology for modeling the effects of command and control on the Forward Air Defense (FAD). It is modeled from the Soviet perspective to judge the effectiveness of the defense against a US penetrating force.

Three possible Soviet defenses which might occur in the FAD are postulated: a SUAWACS, a super AI or autonomous operation. The first represents a best case defense where interceptors operate under the close control of the Soviet Union Airborne Warning and Control System (SUAWACS). The worst case postulated is an autonomous interceptor force which must depend on their own limited capabilities to detect penetrators. The Soviet Union has a large number of interceptors with a variety of capabilities. An intermediate position which would provide some degree of command and control is a super AI. A super AI is an interceptor with advanced capabilities that could be used to control a small number of interceptors.

A analytic, expected value methodology is developed and then adapted to the various command and control defenses described. The measure of effectiveness used is the expected number of survivors. The model provides a number of inputs which can be changed to determine parameter sensitivity. The model is based on

a grid structure to develop the geometry involved in intercepts, similar to that used in the COPEM model. This is one of the first models to explicitly model a parameter for control channels. Another parameter used to capture differences in command and control is the C^2 cycle time.

CHAPTER I

INTRODUCTION

The United States maintains strategic and general purpose forces to provide deterrent and warfighting capabilities. A goal for strategic forces is to survive a first strike and retain the ability to destroy 70% of the enemy's economic, industrial and military targets (Ref 15:21). The TRIAD concept was developed to provide a range of second strike responses which minimized the probability that all US systems could be compromised by technological tools or surprise. The three elements which compose the TRIAD, ICBMs, SLBMs, and the manned bomber, differ in prelaunch survivability, reliability, and responsiveness. They are independent systems whose different characteristics complement each other to provide stability and flexibility in response.

Intercontintental Ballistic Missiles (ICBMs) are highly accurate, readily retargeted, and virtually assure penetration. Sea Launched Ballistic Missiles (SLBMs) are highly survivable and able to endure for long periods at sea, thus providing a high degree of deterrence. The manned bomber brings a great deal of flexibility to the TRIAD since it is capable of changing targets and flight paths. It can hit targets which are mobile, imprecisely located or otherwise transient. (Ref 21:120). Based on the strong points of each of the three elements: time urgent kill capabilities of the ICBM, survivability of SLBMs and the flexibility of the bomber, the TRIAD provides a force which can respond to a variety of threats.

The bomber mission has been modeled in a plethora of ways, using a variety of techniques and assumptions. For the mission to be successful, the bomber must complete many phases. The bombers must first escape the base before a strike hits them on the ground or as they leave. Once airborne, the bombers must receive aerial refueling from tankers which have escaped from their own bases, in order to continue their mission. As the bombers approach the Soviet Union they encounter a number of defensive layers. These include forward air defenses, barrier surface-to-air missiles (SAMs), interval air defenses and terminal SAMs.

Recent advances on both sides have brought into focus the issue of forward air defense.. Until recently, the US penetrating force was composed solely of bombers. The introduction of the air launched cruise missile (ALCM) changed the composition and tactics of the penetrating force. The manned bombers which once were to be penetrating elements could now become a standoff force, stopping at some barrier outside the range of enemy defenses to release the cruise missiles. Alternatively, some other airframe could be a standoff force to deliver the cruise missiles and the manned bombers could remain as penetrators with the cruise missiles.

As a result, the Soviets are reexamining the effectiveness and capabilities of their defenses. Experts agree the issue of command and control has become a deciding factor in conflict. The possible keys to an improved Soviet defense address command and control aspects of the problem: the Soviet Union Airborne Warning and Control System (SUAWACS) and the improved radar capabilities

of their interceptors. The SUAWACS was intended to provide a low-altitude detection capability and the ability to vector fighters to these targets. The radar capabilities of the interceptors are being improved to include a look-down/shoot-down system to identify and track targets against ground clutter. These two capabilities considerably reduce the threat from low altitude penetrators.

A decade ago, the Soviets began work on an airborne warning and control system, the SUAWACS, which is designated by NATO as the Tupolev TU-126 or MOSS. It is thought by US defense experts, to be of limited effectiveness over water and ineffective over land. (Ref 19:102) Recent activity indicates the Soviets are upgrading the Soviet Union AWACS (SUAWACS) using the IL-76 Candid as a mainframe. As many as 30 SUAWACS built on the IL-76 are expected to be operational by the mid-1980's. (Ref 19:102) The shortcomings of the radar which made it ineffective over water should be resolved in this updated version.

The questions of availability and survivability of the SUAWACS have become important issues in the discussion of air defense. Conventional forces may be able to destroy some of the SUAWACS force on the ground or during launch and escape. Destruction of SUAWACS is possible with the advanced strategic air launched cruise missile (ASALM) currently being researched by the US.

As a result, the forward air defense may be operating with or without SUAWACS support. How the loss of a SUAWACS changes the effectiveness of the defense and how to counter such a loss with

available resources emerges as an important consideration. Increased command and control, in this case the SUAWACS, has been synonomous with increased effectiveness. Few studies have tried to measure how much increased command and control improves effectiveness. This study looks at three types of command and control environments which may be used in forward air defense, to identify key variables, assumptions and uncertainties. These may be uniform across the scenarios or may differ among the three. Differences which emerge often highlight subtle interactions or sensitivities of parameters. Once the important parameters of a scenario are identified, the defense can exploit its resources to be as effective as possible under its C^2 environment.

The Foreign Technology Division (FTD) at Wright Patterson AFB was interested in a model which compared the results of Soviet air defenses operating with and without a SUAWACS. They hoped to be able to identify parameters which might critically affect air defense and determine any breakpoints where an increase in capability might strongly increase defense effectiveness. Information obtained from this study could influence the development of future models, and provides background knowledge on the interactions of air defense which may be input to other models or used in studying possible future investment strategies to make the defense insensitive to changes in command and control.

Format

Chapter II summarizes the background research accomplished before building the model. It was accomplished in two phases:

the first was a review of models to identify appropriate techniques and look at the scope of past work in this area; the second phase reviewed the open literature to establish the capabilities of the US penetrating force and the Soviet forward air defense, and identify important parameters for inclusion in the model.

Chapter III presents the general model which provides the basic framework. This model is then expanded as appropriate for each of the three C^2 environments considered.

Chapter IV examines application of the model and considerations in developing input parameters.

Chapter V presents the base case and definition of the parameters which were examined in the model. The results are presented and analyzed.

Chapter VI is a summary of the conclusions which can be made from the results of the model. It also examines some of the limitations of the model and possible areas for improvement and future research.

CHAPTER II

BACKGROUND RESEARCH

Research was accomplished addressing two areas: one was a review of current models of bomber penetration, the other was a review of the current capabilities of the US and the Soviet Union. The review of current models was done to examine the scope of existing models and identify techniques suited for modeling several C^2 scenarios. The capabilities of both sides were reviewed to identify the important parameters which should be included in a model and which warrant the flexibility of being inputs.

Review of Current Models

It is desirable to build on past work when possible, so a review of current models of forward air defense and bomber penetration was undertaken. In examining these models there are two important criteria: the model must have the potential to deal with differing command and control philosophies, and the model must deal with parameters important to command and control such as saturation and interceptor availability. The models listed below are some of the most well known and represent the variety of techniques used in modeling forward air defense.

APM. One of the first models developed was the Advanced Penetration Model (APM) commissioned in 1969 and completed in 1973

by the Boeing Corporation for Headquarters USAF. It encompasses all aspects of the bomber mission: takeoff, base escape, refueling, forward air defense, SAM zones (random, barrier and terminal), terminal and point defenses, weapon delivery at target and recovery at a friendly base. The APM consists of two major segments. The Mission Planner is used to define the overall scenario and then generate individual flight plans. Constraints are imposed on fuel, altitude, airspeeds, etc. The Air Battle Simulator implements, in time sequence, the events which have been specified in the mission planner and inserts others as required by deterministic or probabilistic event assignment. A time history tape of each event is produced providing a wealth of information.

This level of detail allows the user to examine defense saturation, command and control limitations and weapon assignment policies (Ref 10:IV-4). The strengths of the model are the flexibility which the user has via the input parameters which allow one to specify a variety of scenarios. The detailed output listing can be reduced to yield the specific data of interest to the user. This strength is also the cause of the APM's weakness. An implementation of APM is costly both in terms of computer time and real time. This limits its availability for multiple replications and excursions.

SPEED The Simulation of Penetrators Encountering Extensive Defenses (SPEED) model was developed by Calspan Corporation in response to the need to develop a fast running, smaller scale version of the APM. It is a time-sequenced, event-based Monte

Carlo simulation of the penetration of the forward air defense. One of its strengths is that it is a faster but still detailed model.

The offensive elements include manned bombers, guided missiles, drones, decoys and short range missiles and bombs. The defensive elements include a C^2 netting of early warning (EW) and ground control intercept (CGI) radars which report to subcontrol or filter centers and feed to a zone operations center (ZOC). The ZOC pairs penetrators with interceptors. AWACS aircraft orbits between two points at an altitude defined by the user. It assigns interceptors from its own combat air patrol (CAP) to the penetrators it detects.

The SPEED model addresses only the portion of the bomber mission dealing with the forward air defense. This model addresses the efficiency of the interceptor by the percentage of time bombers are engaged by interceptors and the percentage of time bombers are not engaged because of time delays in defense C^2 systems, ECM effects, or saturation of the communication channels or the interceptors. (Ref 16:1-12) A weakness is the number and detail of the parameters which the user must provide.

FISCHER or STRAT DEFENDER. This model was developed by Mr. William Fischer of the North American Aerospace Defense Command (NORAD). It was orginally used to examine the United States Air Defense. It has since been modified and renamed STRAT DEFENDER by AF/SD (Strategic Aerospace Defenses Division) HQ USAF. This simulation model addresses forward air defenses involving unarmed

penetrators against fighters and SAM defenses. The penetrators or raid members are described by their radar cross-section, speed, altitudes and other features. The defense consists of interceptors which have a variety of detection and armament capabilities. A penetrator is liable to interception when it passes within the detection range of a search radar - either GCI or an AWACS aircraft. Interceptors are then vectored from air bases or combat air patrol (CAP) to the penetrator. Detection, conversion and kill actions are determined stochastically. This model also includes other aspects such as location of air bases and their effect on interceptor fuel and armament requirements, SAM sites and their missile capabilities as well as acquisition and committment for various radars.

<u>STRAT PATROLLER</u>. This is an event-based simulation of interceptors flying air surveillance by General Research Corporation for AF/SD, HQ USAF in 1979. It is intended eventually to provide an autonomous interceptor modeling facility to the FISCHER model (Ref 12:2-1). The major focus of the model is the detection function of an air-surveillance barrier. Each offensive element is modelled individually requiring detailed information on flight path, radar, IR and visual observability. Interceptor operation on each leg of its orbit, may vary in velocity, radar mode, sensor scan control, or visual surveillance time. Environmental conditions, lighting, atmosphere and ground radar reflectivity are also considered. STRAT PATROLER currently does not include location of airbases, the effects of fuel, velocity

and range limits on the interceptors, tracking or attack of penetrators, AWACS or GCI surveillance, and IFF and C^3 considerations.

The strength of the STRAT DEFENDER and STRAT PATROLLER combination is that it will provide alternative methods of modeling the defense. Unfortunately, STRAT PATROLLER is still in development, and the links between the models have not been thoroughly established. Another weakness is the amount of detail required, especially by STRAT PATROLLER.

PENEX. This is an analytic model which provides a simplified abstraction of reality rather than the detailed imitation attempted in simulations. Using principles of probability theory, it calculates the expected number of bombers surviving a many-on-many air battle with manned interceptors. No SAM defenses are included in the model. This encounter takes place in a corridor and the air battle is discretized into sub-battles. The penetrators have identical performance and capability parameters. The bombers may carry decoys to be released along the way and neither bombers nor decoys will fire at the interceptors. The interceptors are also identical to one another and they can not distinguish between bomber and decoys.

This model does provide for two different types of command and control philosophies: raid control and close control. In a raid control environment, the general location of the penetration is known and the fighters are sent as a group to randomly search the area, independent of one another. In a close-control

environment, the accurate vectoring capabilities of either a GCI or AWACS result in a uniform assignment of interceptors to penetrators. For each sub-battle, which may use different C^2 philosophies, or weapon loads, the model provides the expected number of bombers surviving, and the expected number of decoys surviving. This output becomes the input for the following sub-battle. A strength of the PENEX model is that it allows two C^2 philosophies to be modeled. The biggest weakness is the abstraction of the situation does not explicitly model many of the C^2 parameters. Specifically, it does not model the interceptor availability as a function of time or the number of control channels available.

COLLIDE. This is an analytic penetration model of air-to-air combat which was developed by Decision Science Applications in 1972 for AF/SD HQ USAF. It is composed of four submodels: a detection model which represents the process of the interceptor detecting the bomber or the bomber detecting the interceptor; a conversion model which maps out the regions of space from which the interceptor can successfully initiate conversion; a command and control model, which introduces the influence of command and control on the conversion process, and an ECM model (Ref 14:11).

It is concerned with finding the probability of detection, conversion and kill based on a one-on-one encounter. It assumes an elliptical detection envelope exists around the bomber and considers the effects of altitude differences, different types of

sensors (infrared and doppler), relative velocities of the aircraft, angularly dependent probabilities of kill and ECM effects when computing the probability of detection, conversion and kill for an engagement.

The C^2 module represents a command and control system, GCI or AWACS, which is able to position the interceptor with known error, at a given position and heading with respect to the bomber. Once positioned the intercept occurs autonomously. (Ref 14:6).

The strengths of the COLLIDE model is the detail provided and its capacity to account for C^2, ECM detection and conversion. However, it only models a one-on-one encounter which makes it a weak candidate for expansion into a FAD model. But, it does provide valuable input data for FAD models.

<u>COPEM</u>. This model was developed at Stanford Research Institute (SRI) as part of a study to improve the representation of airborne strategic systems in aggregated effectiveness evaluation models (Ref 7:v).

This model generates average penetration probabilities as a function of penetration depth. Penetration is defined as the depth, d, the penetrator travels into a single attack corridor. The corridor is modeled using a grid which serves as a frame of reference for the positions of the penetrator and the interceptor and is used to compute interception times. The relationship between distance, velocity and time allows a substitution of variables which gives the average penetration probabilities as a function of time if a constant velocity is assumed.

A fundamental assumption of the model is that the number of intercepts which can be made on a bomber follows a Poisson distribution with a time dependent parameter. (Ref 7:6). This parameter incorporates all information about the interceptor force which influences the number of interceptors airborne and available for asssignment. The parameter is estimated iteratively for discrete time intervals.

COPEM's strengths include the variety of parameters which are modeled and their flexibility. Its weaknesses are found in some of the assumptions which limit its applicability to forward air defense scenarios. This model was examined in detail as a candidate for modification to represent C^2 effects in forward air defense. In examining and trying to apply the model, some assumptions which limit its applicability were discovered. The model is presented in detail in Appendix A and the implication of these assumption discussed.

<u>Summary</u>. There are numerous bomber penetration models using a variety of methods. Some bomber penetration models encompass all aspects of the mission in differing levels of detail, other models, just the air defense portion including interceptors, or SAMS, or various combinations of the two.

There essentially two methodologies used to model air defense: simulation and analytical techniques. Simulation models are often very detailed and complex. This wealth of detail allows much information to be accumulated for output, and allows many situations to be modeled with the flexibility of the inputs. Unfortunately, this detailed analysis is the greatest weakness of

simulation models since it requires a large amount of research for input data and usually dictates a long running model. Thus, detailed outputs may not be in a convenient form for the use, requiring a separate excursion into report writer programs to examine particular parameters. Many simulation models have grown so large that no single individual understands the full program. An example of such a model is the APM.

Analytic models usually lack the detail of simulations. They often make assumptions which simplify reality but keep the mathematics tractable. These models are advantageous since they often require fewer inputs and are relatively quick to calculate. As a result, the solutions are simple, easy to understand, and can be examined for relationships between elements in the model. Hence, synergisms can be recognized and exploited when analyzing tactics and policies.

No single model has been developed which can be applied to several types of C^2 situations. Simulations often involve very detailed inputs and are scenario specific, analytic models usually have wider applicability. A more detailed analytic model is desired since it can provide flexibility of C^2 parameter inputs, without requiring an abundance of input. Once an underlying foundation of principles for the model is developed, it can be modified to represent other C^2 deployments. The question and issues to be considered with this study involve relative changes rather than addressing absolute answers.

Assessing the Capabilities

Before developing a model it is important to identify what parameters should be examined and how much flexibility is required in the model. An assessment of what is operationally feasible can prevent nonsensical models and model results which do not accurately reflect reality. This section discusses Soviet concepts of air defense and focuses specifically on forward air defense. It also addresses the "threat" which, as seen by the Soviets, is the US penetrating force. US technological advances, such as the B-1B bomber and the advanced technology "Stealth" bomber may substantially change the composition and type of penetration force, but their acquisition and production levels are uncertain due to the political intricacies of funding. Soviet advances are fielded rapidly so their force tends to be a mixture of old and new. Hence, this study only considers the near future, encompassing the mid 1980's through the early 1990's. Beyond that, technological advances and acquisition decisions may invalidate the results due to changes in force composition, tactics, etc.

US Penetration Threat Composition. The backbone of the US bomber force is the B-52, an aging aircraft built in the technology of the 1950's. There are a variety of types based on modifications which have been made including: 75 operational B-52Ds, 151 B-52Gs, and 90 B-52Hs. Currently, they are undergoing a modernization program to enhance their effectiveness. The B-52G/Hs will be outfitted with a new offensive avionic system to

upgrade the navigation and weapon system delivery. In addition, B-52s will be hardened against electromagnetic pulse and outfitted with improved electronic countermeasures (ECM) equipment. (Ref 21:III-61). The B-52G/H will be adapted as the carrier aircraft for the cruise missile. Initial operational capability scheduled for December 1982, will modify the aircraft to carry 12 ALCMs externally, 6 under each wing. Later in the 1980's the aircraft will be modified to carry 8 more internally. The B-52G/H can fly 6,513/8687 nm without refueling. Its cruising speed at high altitude is Mach .77 and penetration speed at low altitude is Mach .53 to .55 (352-365 knots) (Ref 19:299).

The cruise missile is a threat which causes great concern for planners of forward air defense. It is a small, unmanned winged air vehicle capabable of sustained subsonic flight following launch from a carrier aircraft. It has a smaller radar signature than a B-52 and is capable of low-altitude penetration. The range of the cruise missile is currently planned to be 1350 km, but a range of 2000 km is easily achieved. The protocol to the SALT-2 agreement contains a range limitation of 2000 km. (Ref 17:16).

The cruise missile would allow carrier aircraft to act in a standoff role, returning home after launching its load at some barrier. The limited range of the cruise missile dictates the location of the standoff points relative to the targets inside the country. It is possible the Soviets will establish a barrier defense at these locations using the SUAWACS and long range interceptors. This force could impose a standoff range approximately 400 nm from the coast line. The cruise missiles

launched at this point would only be able to reach about half of the missile field targets and about 80% of the industrial targets. The use of tanker aircraft or the long range MIG-25 Foxbat interceptors could increase this standoff range. (Ref 17:19).

Target of the US Penetrating Force. A retaliatory strike by the US has a goal of destroying a large portion of the Soviet military, industrial and economic strength. Certain key areas have been identified as being probable target areas. These are shown in Figure 1: Moscow, Leningrad, Donets, Urals and Baku. Four other areas are of secondary importance: Tashkent, Kuznetsk, Baikal and Vladivostok. In the first five araas: Moscow is the capital and nerve center of the country; Leningrad is the center of culture, science and tehcnology; Donets is a key industrial area and power source; the Ural and Baku provide crucial raw materials such as iron ore, petroleum and oil, as well as having industrial reserves. (Ref 6:130-2)

Approach of the US Penetration Threat. There are numerous SAC bases located across the US. One third of the US bomber force is on alert at any moment ready to take off within minutes. (Ref 3:7). This ensures that some bombers will be able to launch and survive a pre-emptive strike by the Soviet Union. The bombers act independently but all fit into the master plan, called the single integrated operational plan (SIOP). The details are carefully guarded but planners try to ensure that the bombers that survive penetrate to key areas. Fuel restrictions dictate the bombers fly

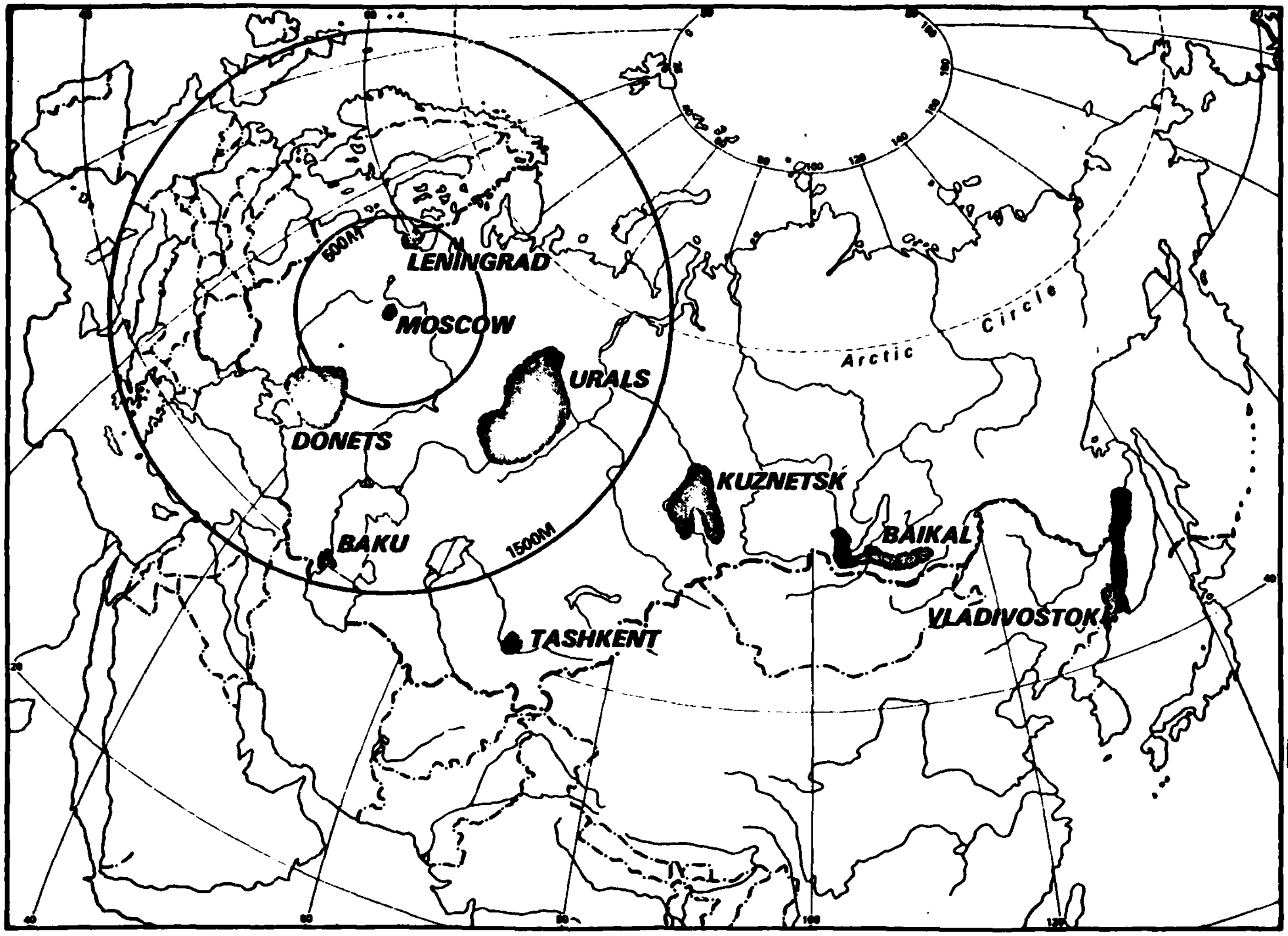

Figure 1. Penetration Corridors

at high altitudes using a fairly direct route over the North Pole into fairly narrow corridors, and then use low altitude penetration and some deceptive manuveurs when they enter the defense of the country.

Soviet Forward Air Defense. To counter the capabilities and intentions of the US threat, the Soviets have given considerable attention to air defense. In general, Soviet air defense falls into one of two categories: a forward air defense which exists close to the coast of the USSR to attack and kill ALCMs or bombers before they reach the land mass of the USSR, and in-country air defenses which use both interceptors and surface to air missiles. As mentioned, Soviet Air defense may manifest a third face, a long range or barrier air defense whose purpose is to intercept the cruise missile carrier aircraft before they have a chance to release their cruise missiles.

Importance of FAD. Forward air defense is of current interest since it is there that as many cruise missiles as possible must be destroyed. The long range defense has yet to be developed, so for the immediate future, the effectiveness of the forward air defense is crucial. In the past the Soviets faced a relatively sparse target environment where locating the targets was difficult. In addition, low altitude penetration made it even harder. Close control and efficient use of airborne interceptors (AI) could be critical. The SUAWACS provides low altitude radar coverage and can be used to vector the interceptors or pass

commands to GCI facilities for vectoring.

The Soviets have many GCI radar sites which may be comparable to the US DEW line radar. These radar have a high altitude detection range of 200 nm but are useless against low altitude penetrations until the penetrators are close to land, 40 - 50 nm. (Ref 6:162).

SUAWACS. The SUAWACS is being improved using the Candid aircraft and will be assumed to have capabilities similar to the US AWACS for the 1985-1990 time frame. The AWACS ceiling is 29,000 ft., its endurance on station 870 nm from base and its maxlevel speed is 460 knots. It can move at Mach .8 and stay airborne 10 hours or longer. (Ref 20:44) Its crew of 17 includes 13 AWACS specialists; this number can vary for tactical and defense operations. Pulse Doppler technology with high repetition frequency allows the AWACS to provide an early warning airborne center for identification and tracking of high-or-low level penetrators in all kinds of weather over all kinds of terrain. It acts as a command and control center for air defense forces. The AWACS coverage extends more than 200 nm for low-flying penetrators and even further for high altitudes. (Ref 19:298-9)

Soviet Interceptor Force. The Soviets have a wide variety of interceptors available to their air defense which feature a variety of capabilities. Location of Soviet air bases and interceptors is not readily available from the open literature,

but there is considerable discussion of dispersal bases available for fighters to fly to before ICBM or SLBMs hit, to increase survivability. The most like candidates for the forward air defense mission are the MIG-23, MIG-25 and FIDDLER aircraft. The maximum combat radius of these fighters ranges between 300-900 miles. This includes takeoff, climbing to altitude, flying a fast cruise speed to a position, 3-8 minutes of intercept at high speed and return to base at endurance speed. The radar detection capabilities include search radar of 18 to 55 miles and tracking radar in the 8 to 35 mile range. (Ref 20:98-100) These interceptors generally carry 2 or 4 air-to-air missiles (AAMs) known as Aphid, Apex, Acrid, or ATol I. These are either radar or infra-red (IR) guided with ranges of 3 to 20 miles. (Ref 19:109)

Possible Scenarios. As discussed earlier, the general direction of approach of the US bomber is known to the Soviets who will probably deploy the SUAWACS to provide early warning in these area. Information acquired on size, location and direction of the penetrating force is sent to some regional HQ for decision by higher authority. Since the interceptors are limited in range, and in general have a short endurance time (1-2hrs), they must be used efficiently. The SUAWACS radar coverage can move with the penetrators, allowing it to continue to relay information on the threat size and direction. From this data, the time and location when the penetrators will come within range of the interceptors can be estimated. The fighters can be scrambled and enter SUAWACS control for the interception attempt. The number of interceptors

assigned to each penetrator will be dependent on the number available. Although the Soviets have a large force, 2700 aircraft total, many may not survive or escape to dispersal bases. In case the SUAWACS is unavailable or is shot down, the interceptors will be left to operate autonomously. Assuming some type of early warning, either from the AWACS or some other system, probable arrival time and direction of the penetrators can still be estimated and the interceptors can be scrambled. However, the interceptors must then depend on their own limited radar and detection systems to located and destroy penetrators.

One possible stop gap measure restoring some degree of command and control, is the use of a super AI. The Soviets have a large interceptor force with a variety of capabilities; a more advanced AI, a "super" AI could become a control center for the other AIs. Its detection capability could be far beyond the other AIs, allowing it to make more efficient use of these resources than would occur autonomously. Still, the super AI remains considerably more limited than the SUAWACS in the number of interceptors it can control and in its detection capabilities.

<u>Summary</u>. Some of the shortcomings of autonomous interceptors in air defense are obvious. The interceptor radar is susceptible to jamming and they lack the range and altitude determination capability of the SUAWACS. When the efficiency of the command and control of the SUAWACS is lost, two alternatives appear: some degree of C^2 can be restored through the use of a super AI, or autonomous operations must be conducted efficiently.

Discovering which parameters are crucial in a reduced C^2 scenario is important, not only for finding ways to make the defense better, but in bringing a better understanding of the synergism of the elements in FAD and how command and control contributes to improved effectiveness.

CHAPTER III

MODEL DEVELOPMENT

This section presents a set of models of various air defense scenarios, founded on a single set of underlying principles which is then expanded as appropriate for each scenario. The fundamental model addresses the problem of a group of penetrators passing through the forward air defense zone which is protected by a group of airborne interceptors. The model finds the expected number of penetrators which survive the forward air defense for a variety of scenarios as defined by the user. The specific main assumptions used to define the problem are given below.

The forward air defense coverage zone is represented as a corridor defined by its width and depth, that the defense wants to protect. It is assumed to take place out over water beyond the range of ground control radars and it does not contain any targets. Geography is specified by a two-dimensional grid used to locate the penetrators and compute intercept times (see Figures 1 and 2).

The penetrators choose an entry point at random along the boundary. The FAD coverage zone is divided into columns to represent the possible entry points a penetrator may choose. A penetrator is assumed to enter at the center of the column and fly through the corridor in straight lines parallel to the sides of

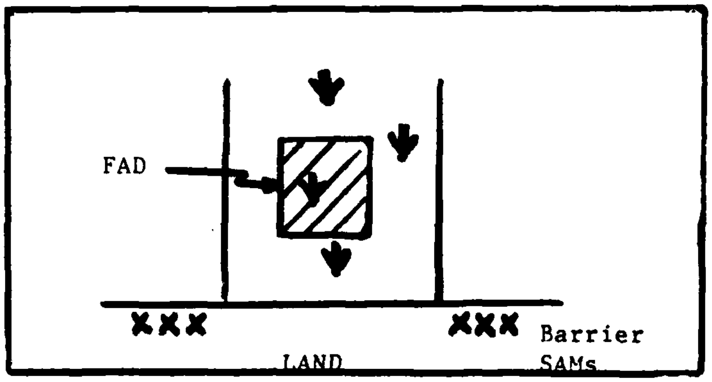

Figure 1. Penetration Corridor

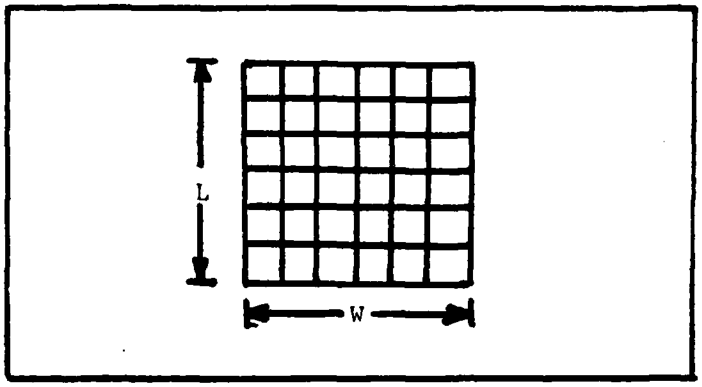

Figure 2. Two-Dimensional Grid of FAD Coverage

the grid. The penetrators arrive at a constant rate over some period of time, referred to as the entry window. Thus, the interarrival rate is determined by the number of penetrators which arrive during the entry window.

Detection of the penetrators and subsequent fighter assignment is a function of the command and control system used in the scenario. The three possible defenses which will be considered are shown in Figure 3.

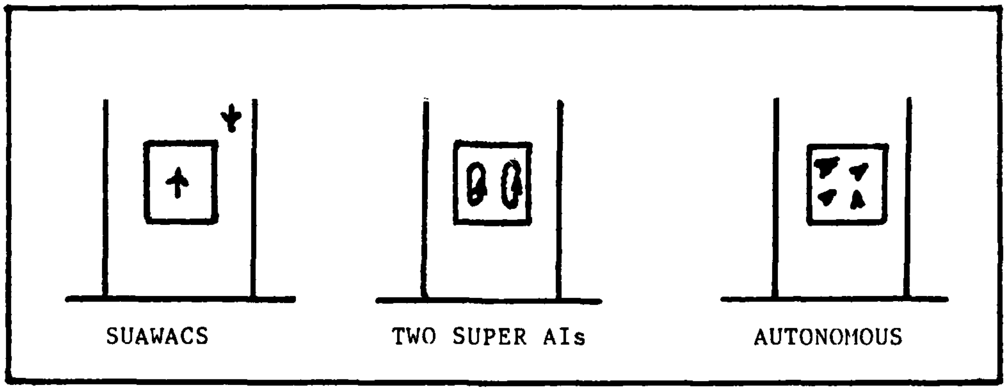

Figure 3. Possible Defense Scenarios

As shown, the FAD coverage by the defense may not encompass the entire corridor. Normally, the effectiveness of the defense is based only on those penetrators which enter the FAD coverage. Penetrators which enter outside the coverage are ignored, such as penetrator A. If considering a single model only the effect of the defense on the penetrators which enter the coverage zone is considered. When comparing models, as in this study, some base case coverage zone must be established. The effectiveness of the defense for this base case is assessed for each model. If the actual coverage zone of a defense is smaller than the base case coverage zone, then the penetrators which enter outside the actual zone but inside the base zone must be added to the number surviving the actual coverage of the defense. In this study, the base case is the coverage zone of the SUAWACS. For example, let the SUAWACS coverage be represented by a grid of 500 miles by 500 miles. Now consider autonomous interceptors whose coverage zone has a length of 250, and width of 400. Penetrators such as A and B (Figure 4) which enter inside the SUAWACS coverage but outside

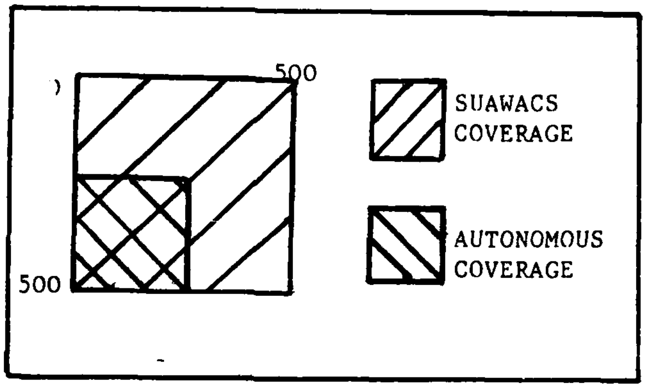

Figure 4. Autonomous vs SUAWACS Coverage

the autonomous coverage, must be added to the survivors of the autonomous coverage to allow valid comparison of the two models.

Some type of early warning is assumed to reach the defense, so the interceptor force can be scrambled and ready when the penetrators arrive. It is assumed the AIs do not need to return to refuel during the penetration. The length of the penetration begins when the first penetrator enters the FAD and ends when the last penetrator exits. The AIs are capable of remaining airborne without refueling for this length of time; however, they do need to return to base to rearm.

An AI assigned to a penetrator has enough weapons for two shots. A shot may consist of one or more weapons. For example, if a shot consists of two missiles, the interceptor has a total of four missiles, two for each of the two shots. The single shot probability of kill the user provides should reflect the kill probability of a shot as defined here. The AI may kill the penetrator on the first shot and will then be available for reassignment to one more penetrator. Or the AI may miss the penetrator on the first shot and will then use his remaining shot

for a second attack. Once an AI has used all his weapons, he must return to base to rearm. The AI is replaced by another AI after some delay. This delay is the replenishment time and it remains constant for the length of the penetration. The interceptor is unavailable for the length of time required for the intercept. Limitations may also exist on the number of fighters a control system can handle simultaneously. Although there may be enough AIs available to assign to all the penetrators detected, the C^2 system may be limited to making only a portion of the intercepts by its control capacity.

Other assumptions of this model which are self-explanatory are:

1. The penetrator force is homogeneous.
2. The penetrators do not fire at the AIs.
3. The penetrators travel at the same constant speed.
4. Attrition of the penetrators is due solely to AIs.
5. The interceptor force is homogeneous.
6. The interceptors are assigned to each of the penetrators with equal probability.

The assumptions are summarized in Tables 3 and 4 at the end of this chapter.

Overview

A penetrator flying through the forward air defense zone survives if he is either undetected or is detected but survives all attacks. The defense must try to destroy these penetrators by detecting the penetrators and then making interceptor assignments.

An assignment as made by close control models is an assignment of an AI by the C^2 system. In the autonomous case, an AI which makes a detection assigns itself to make intercept attempts. The time of detection, relative to the penetrator's entry into the FAD partially determines how effective the defense can be. The availability of interceptors is another critical factor in the effectiveness of the defense. If interceptors are not available when penetrators are detected, then assignment cannot be made at this time.

The defensive command and control system determines when detection occurs and how interceptor assignments are made. In close control situation, once a penetrator is detected, the location of the penetrator is known from that point until he leaves the FAD. In autonomous operations, each interceptor must detect and then make the attack. If the attempt fails, then the next interceptor the penetrator encounters must again detect the penetrator before making an intercept attempt.

At any point in time, the number of penetrators located in the corridor is composed of all those surviving. The command and control system detects and makes interceptor assignments against those penetrators. The percentage of the penetrators which receive interceptor assignment is dependent on interceptor availability and control capability of the C^2 system. If there are more interceptors than penetrators requiring assignment, then an interceptor can be assigned to each penetrator requiring assignment. Otherwise, all the interceptors are assigned and equally distributed among those penetrators requiring assignment.

In the latter case, some penetrators will not be assigned interceptors.

A further limitation in the close control case is the number of interceptors the C^2 system can control, or number of control channels. If the number of control channels is less than the number of AIs available, then the number of control channels becomes the bound on the number assigned. Of course, if the number of AIs available is less than the number of control channels, the number of AIs available becomes the bound. Thus, the number of interceptors assigned is the minimum of the number of interceptors available, the number of interceptors the system can control, and the number of penetrators needing assignment. Only one AI is assigned to a penetrator even if the number of AIs available is greater than the number of penetrators needing assignment.

In making an intercept attempt, time is required for the C^2 system to decide on a course of action, (T_A), for the interceptor which is assigned to fly to the penetrator, (T_F), for the AI to search and acquire the penetrator, (T_S), and for the AI to convert and launch a missile, (T_L). Three of these, T_A, T_S, T_L, are essentially constants; T_F is the only one dependent on the positions of the penetrator and the fighters on the grid. Since both the penetrator and the AI are assumed to be located in the center of their respective cells, T_F can be calculated using the geometry of the situation. The interceptor is flying either at the CAP or autonomously searching at a loiter velocity to conserve fuel. In making the intercept attempt, the

AI flies at a much higher speed, dash velocity to the penetrator. This derivation is given in Appendix B.

In the autonomous model, the penetrator and interceptor are collocated and flying time, T_F, is zero. However, in the close control models, assignments are made from a CAP position located in the center of the coverage. The time to fly to a penetrator, T_F, is computed from this position to the penetrator's position. If the AI is successful on the first shot, he requires an additional amount of time, T_F, to return to the CAP position before he is again available for assignment. If the AI uses both shots in his attack, a new AI is sent to replace him requiring T_R, the replenishment time. So this AI is not available for assignment until the replacement arrives. Note that the number of control channels used in the intercept becomes available immediately after the intercept is complete.

Since each AI has two shots, it may use one or both, in the intercept attempt. In all scenarios, the interceptor has a shoot-look-shoot capability, the AI does not fire a second shot unless the first one fails. Rather than keeping track individually of the number of interceptors with one shot left and those with no shots left, the concept of equivalent interceptors was used. This retains the homogeneity of the interceptor force, all AIs available for assignment have enough weapons for two shots. After assignment and attack by the AIs, the number of interceptors with one shot left is divided by two, and this number of equivalent interceptors is returned to the interceptor force. Suppose ten AIs are assigned with a single shot kill probability

of .6. Then four AIs will have one shot left. These four AIs with one shot left can be represented by two AIs with two shots. Thus the equivalent number of AIs returned to the interceptor pool after the intercept is two.

As interceptors are assigned to penetrators, the number of interceptors available for assignment is reduced until the interceptors either return or are replaced. The AIs with 1 shot left return to the force, AIs which use both shots are replaced by another interceptor. Interceptor availability is a function of time and is depleted as interceptors are assigned, and incremented as AIs return or are replaced. The frequency of assignment is a function of the command and control system.

Although a C^2 system is continuously making assignments as AIs become available and penetrators are detected, this model makes all the assignments for a time interval at discrete points. The size of time intervals is a function of the C^2 system and is discussed later for the specific defense scenarios. If the time interval is five minutes long, then at $t = 5$ the C^2 system makes all the AI assignments against penetrators detected at from zero to five minutes.

At each interval, assignments are made and the number of penetrators is modified to account for kills. Figure 5 shows the events which occur at time interval m; Figure 6 shows the results.

At time interval M:

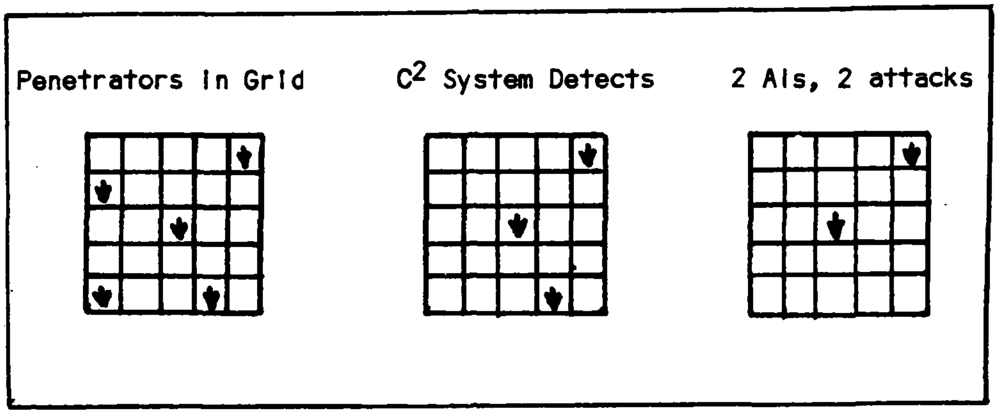

Figure 5. C^2 System Assignment with a PK = .5

At next time interval M+1, all penetrators fly forward 1 row and average number of penetrators in grid:

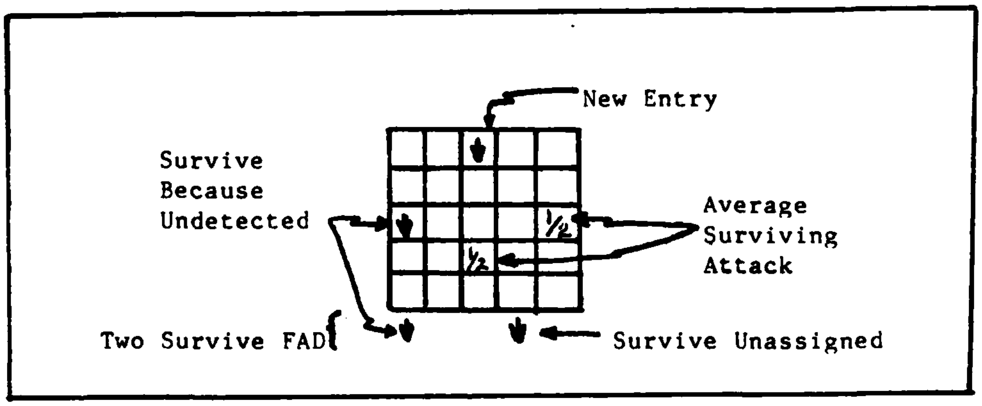

Figure 6. Penetrators Surviving C^2 Assignment

Some of the penetrators survive because they were undetected, some are detected but do not have an interceptor assigned to attack them due to unavailability of the AIs, and some survive the attack made on them. In the next interval, new penetrators enter, and some of the surviving penetrators now exit the forward air defense zone. By accounting for entries, exits and kills, an

estimate of the number of penetrators in the zone at the next time interval is obtained. The number of surviving penetrators which exit the forward air defense zone in each interval can be summed for the length of the penetration to get the expected number of penetrators surviving the FAD.

To summarize, this is an expected value model which calculates the expected number surviving the FAD by summing the survivors which exit the FAD for each discrete time interval of the penetration. The size of the time intervals reflects the capabilities of the C^2 system to make detections and fighter assignment. In each interval, the penetrators in the zone are one of three types: undetected, detected but unassigned, or detected, assigned and surviving. Some of these penetrators are detected by the command and control system and are assigned fighters as interceptor availability permits. Future AI availability is modified to reflect the assignment of AIs at this time interval. Also, the number of penetrators in coverage is modified to account for kills. This continues for the length of time of the penetration.

General Development

Consider a FAD coverage zone of length L, and width W. The defense has a C^2 system which makes AI assignments for time intervals of length Δt. During the first interval, $I(1)$ penetrators arrive at the FAD zone. At the next interval, the penetrators which entered previously fly forward a distance, $v_p \Delta t$, where v_p is the velocity of the penetrator. The length

of the grid is divided into r rows through which the penetrators advance, one row each interval, until they exit. The depth of the row is equal to the distance a penetrator can travel in timeΔt (see Figure 7).

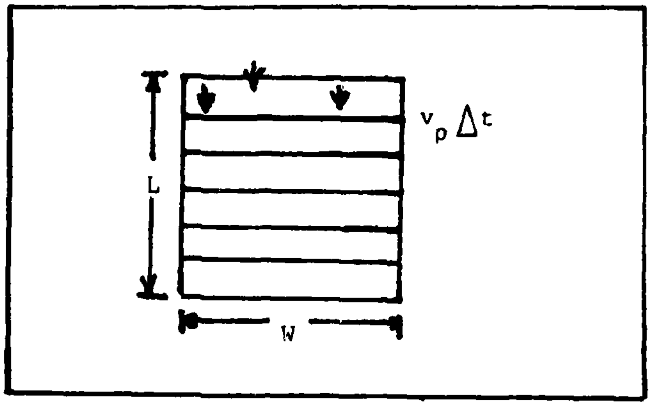

Figure 7. FAD Coverage Zone

Define:

L - length of FAD zone

W - width of FAD zone

Δt - length of interval between successive detections and AI assignment by the C^2 system

m - an index for each of the Δ time intervals that penetrators are in the grid

I(m) - number of penetrators which arrive in each interval to the FAD

$D_i(m)$ - the number of penetrators detected in row i needing AI assignment (i=1,2,...,r) at time interval m

$S_i(m)$ - the number of penetrators surviving previous attacks located in row i needing AI assignment at time interval

m $(i=1,2,\ldots,r)$

$T(m)$ - the total number of penetrators needing AI assignment at time interval m

$NAI(m)$ - the total number of AIs available for assignment at time interval m

NAI_c - the number of AIs the C^2 system can control simultaneously.

$NAI_s(m)$ - the number of AIs which are assigned at time m

For each interval, the number of penetrators detected needing assignment in each row and the number surviving needing assignment in each row, are added together to get the total number needing assignment at this time.

$$T(m) = \sum (D_i(m) + S_i(m)) \qquad (3.1)$$

The number of AIs available at this time is denoted by $NAI(m)$. The number of AIs assigned is the smaller of the total number of penetrators needing assignment, the number of AIs the C^2 system can control, and the number of AIs available, $\min(T(m), NAI(m), NAI_c)$. If the number of interceptors assigned is equal to the total number of penetrators needing assignment, then each penetrator has an AI assigned to it. There may be enough AIs available to assign to each penetrator, but the C^2 system cannot handle that many interceptors simultaneously. If this is the case, the number of AIs assigned is equal to the number the C^2 system can control. If the number of AIs available at time interval m falls below the control capacity of

the C^2 system, and the number of penetrators needing assignment at time interval m exceeds this, all the AIs available are assigned to penetrators.

$$NAI_s(m) = \min(T(m), NAI(m), NAI_c) \tag{3.2}$$

These interceptors will then be occupied for the length of time required to make the intercept and to then return or be replaced. The function for interceptor availability is revised to reflect the future unavailability of these interceptors.

$$NAI(m+1) = NAI(m) - NAI_s(m) \tag{3.3}$$

where m is modified for the necessary length of time until the AIs return or are replaced. If this time is smaller than the time increment Δt, when the next C^2 check is made, the interceptors will return or be replaced before their services are needed again, so no modification is made to the availability function.

The number of penetrators detected needing AI assignment at the next time interval also is modified to subtract those penetrators who had AIs assigned at this time interval. The number surviving at the next time interval is the number surviving who did not have an AI assigned in this time interval. And the number surviving the AI attacks needs to be adjusted to account for the number of penetrators killed. The outcome of the AI attacks may not be known immediately, but occurs upon completion of the intercept. The length of time to make the intercept is

$T_A + T_S + T_L + T_F$. This is translated into the discrete time intervals by:

$$M1 = \text{INTEGER} \quad \frac{T_A + T_S + T_L + T_F}{\Delta t} + 1 \tag{3.4}$$

After this time the C^2 channels become available. It is also the time that AIs with 1 shot return in the autonomous model. In the close control models, the AIs with 1 shot will return after M2 additional time intervals where:

$$M2 = \text{INTEGER} \quad \frac{T_A + T_S + T_L + 2T_F}{\Delta t} + 1 \tag{3.5}$$

And the replacement for AIs which use all their shots arrive after M3 additional time units where:

$$M3 = \text{INTEGER} \quad \frac{T_A + T_S + T_L + T_F + T_R}{\Delta t} + 1 \tag{3.6}$$

The number of kills is known at time interval m + Mi and these surviving penetrators may again be assigned an interceptor. The C^2 system does not place a priority on penetrators, so the AIs assigned are equally distributed among the penetrators needing assignment.

$$D_i(m+1) = D_i(m) - \frac{D_i(m)}{T(m)} NAI_S(m) \qquad i=1,2,\ldots,r \tag{3.7}$$

$$S_i(m+1) = S_i(m) - \frac{S_i(m)}{T(m)} NAI_S(m) \qquad i=1,2,\ldots,r \tag{3.8}$$

$$S_i(m+k) = \frac{D_i(m) + S_i(m)}{T(m)} \quad NAI_s(m) \qquad i=1,2,\ldots,r \tag{3.9}$$

The number of surviving penetrators in the last row, both $S_r(ml)$ and $D_r(m+l)$, at this time interval are added to the total number of penetrators surviving the FAD, since they will exit the FAD before the next C^2 check is made and hence will no longer be liable to attack.

$$TS(m+l) = TS(m) = S_r(m+l) + D_r(m+l) \tag{3.10}$$

All penetrators are shifted forward a row, to prepare for the next time interval. The calculations are done repeatedly for m+l,2,... through all the time intervals of the penetration.

$$D_{i+l}(m+l) = D_i(m+l) \qquad i=1,2,\ldots,r-1 \tag{3.11}$$

$$S_{i+l}(m+l) = S_i(m+l) \qquad i=1,2,\ldots,r-1 \tag{3.12}$$

The number and location of the penetrators which are detected in each interval is not specifically addressed since it is dependent on the command and control system involved. The following sections explore in detail this aspect of the problem and the specific C^2 applications.

Distinctions Between C^2 Systems

This foundation of principles provides a common core for the application of the model to specific scenarios. There are three facets of the model which allow the command and control

capabilities to be specified: the manner of detection, the times involved in making the intercept, and the row size of the grid, or equivalently, the length of time Δt, between C^2 AI assignments.

There are two philosophies of detection represented in these models. In the SUAWACS and super AI models, once a penetrator is detected, he is always detected; knowledge of the penetrator's location is retained. In the autonomous model, when a penetrator is detected, he is detected only for that period Δt. Knowledge of his location is not retained, but must be reacquired in succeeding time periods.

Another distinction addresses <u>when</u> detection occurs relative to the penetrators' time of entry. The model allows detection probability to be specified as a function of the rows, and hence, as a function of distance. For the examples considered in this study, the probability of detection is defined in the following manner. In the SUAWACS model, if detection is going to occur, it occurs when penetrator first enters. In the super AI model detection occurs in any row with equal probability. In the autonomous model it is a recurrent event with a detect/no detect decision made by each AI. The SUAWACS radar coverage is considered more capable and extensive than the super AI. This allows the SUAWACS to make detections in the first row as soon as the penetrators enter the grid, so the penetrators are liable to attack, if the AIs are available, from the time they enter the grid until the time they exit. The super AI has a limited radar, scanning the area in front of him, he requires longer to check the entire grid. Penetrators are detected by the super AI in any one

of the rows with equal probability. Thus they are only liable to attack from the time of detection until they exit the grid. In the Autonomous model, a yes/no decision is made on detection with the number of detections which can be made limited by the number of interceptors. The interceptors are scattered at random through the grid and their number and availability at any position will differ with time and location in the grid as the penetrator flies through.

The second parameter used to distinguish between the C^2 systems is the cycle time for the C^2 system. This parameter attempts to quantify the capability of the C^2 system to make assignments against penetrators. A more capable C^2 system can check the grid and make assignments against penetrators more frequently. Thus, it will have a smaller C^2 cycle time. The SUAWACS essentially checks continuously, since its radar covers the entire grid at all times. The size of Δt can be made very small. As Δt is made smaller, the depth of the row also decreases and the number of rows in the grid increases. Determination of how small to make Δt may also be made relative to the cycle time of the other models. For the super AI with a less capable radar, the cycle time is essentially some average time until it sees the same point again (see Figure 8).

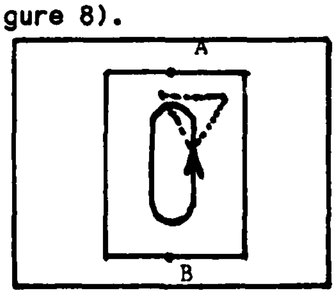

Figure 8. Super AI Search Pattern

The endpoints of its search pattern, points A and B, will only be seen each time the super AI completes an entire cycle. Many points in between are seen much more frequently and do not have to wait for the super AI to complete one search pattern. Some time interval can be estimated which appropriately reflects the super AI's radar capability and search pattern. Many rows indicate a C^2 system which makes frequent checks, few rows indicate a C^2 system which cannot make many checks while a penetrator is in coverage. However, it is the size of the C^2 cycle interval which drives the row size.

The method used to divide the grid into cells in the autonomous case is chosen by the user. For this example and in the remainder of this study, the following method is used. In the autonomous case the grid is divided into cells, each of which reflect the radar coverage at a single AI. The search pattern of an interceptor and its radar capability defines a cell (see Figure 9). If a penetrator flies through this region, then the probability of detection is Pd. The grid is divided into cells which represents the coverage provided by a single AI, and the AIs are divided among these cells. Each time a penetrator enters a row it may possibly be detected and have an intercept attempt made against it by the interceptor in this cell .

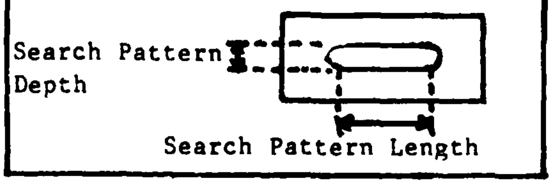

Figure 9. Autonomous Search Pattern

The third aspect of the model which makes distinctions between C^2 systems are the times involved in the intercept. T_A is the data processing time required by the system to confirm the penetrator on its radar. T_S is the time for the interceptor to search and acquire the penetrator on its radar, when in the near vicinity. In the SUAWACS case, the interceptors are vectored by the SUAWACS to the penetrators. This should reduce T_S for the individual AI. The super AI doesn't have as much control over the interceptors, so it will take longer for the attack AIs to search and acquire the penetrator. It also may require a longer T_A, because of less refined capabilities for data processing. In the autonomous case, these times, T_A and T_S, are interleaved in the individual AIs search and attack process, but the combination of the time required is probably greater than those of either the SUAWACS or the super AI.

Table I summarizes the aspects of the model which make the C^2 distinctions.

<u>SUAWACS Close Control Model</u>

One of the best defensive deployments is when the SUAWACS is used to control the interceptor force. The ability to track penetrators and make interceptor assignments allows maximum use of the resources available to the defense. With SUAWACS guidance, the interceptor is better able to find the penetrator, lock on, and fire. Soviet use of interceptors in this manner relies heavily on the SUAWACS to guide the interceptor completely through

TABLE 1

C^2 DISTINCTIONS

Parameter	SUAWACS	Super AI	Autonomous
t	1/min	1/3 min	1/5 min
*P_d	.9	.7	.4
Time for Intercept			
T_A	1.0 min	1.5 min	5.0 min
T_S	2.0 min	2.5 min	
T_L	.5 min	.5 min	.5 min

*An important distinction in the probability of detection among the three models. The Pd for a close control model has a much different application than for the autonomous case. In close control, it is the probability of detection of the penetrators by the control centers; in the autonomous case, it is the probability of detection by the individual AIs.

the intercept. It does not take advantage of advanced interceptor capabilities, it also requires the SUAWACS to be tied up in the intercept the entire time required for the intercept.

The SUAWACS is capable of 360 degree continuous coverage. This provides almost complete coverage over the area the SUAWACS patrols. This study addresses the effectiveness of the SUAWACS over the area it covers. The grid size represents the coverage which can be achieved with a particular system. If the SUAWACS' radar range is limited to 500 miles, and the user defines a penetration corridor of 1000 miles, then only 500 miles will be defined in the grid. Penetrators which enter along the remaining 500 miles will survive since they enter outside the coverage of the SUAWACS (see Figure 10). The area of SUAWACS coverage becomes the base case for model comparison.

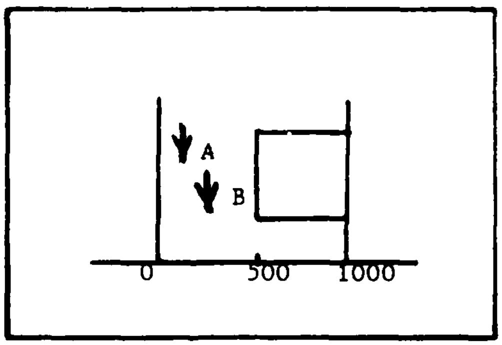

Figure 10. SUAWACS FAD Coverage

The location of the coverage zone in the FAD doesn't matter, since the penetrators' entry points are uniformly distributed. The results of the model will reflect kills against those penetrators which actually enter the SUAWACS coverage. Penetrators such as A and B survive because they do not pass

through the FAD coverage zone.

Detection occurs with probability P_d when the penetrators enter the leading edge of the SUAWACS radar coverage. The number of detections a SUAWACS can handle is unlimited, but other factors such as the numer of control channels, limit his ability to act on the detections. Another aspect of the detection process which should be emphasized is that once a penetrator is detected, his location is known the remainder of the time he is in the FAD.

The probability of detection is expressed as a function of what row the penetrator is in. In this manner, P_d can be expressed as a function of distance, improving the probability of detection when the penetrator is close to the SUAWACS, and lowering it as the penetrator is further away. Another possibility is to allow most of the detections occur immediately in row 1, while the remaining detections occur at random in rows 2 to r.

For this particular study, the assumption was made that if detection occurs, it occurs in the first row. This was used to emphasize the more powerful SUAWACS radar.

$$P_d(i) = \begin{cases} P_d \text{ if } i = 1 \\ \\ 0 \text{ if } i=2,3,\ldots,r \end{cases}$$

where P_d is the probability of detection (user defined), and

$P_d(i)$ is the probability of the detection in row i.

The ability of the SUAWACS to act on detections and make interceptor assignments is limited by the number of intercepts available or the number of control channels. If at time interval m, the SUAWACS detects penetrators, and has no AIs available to the CAP to assign, then the penetrators are not attacked and will survive to the next interval. These penetrators have been detected but are unassigned. At the next time, M+1, these penetrators may again not receive an interceptor assignment. Thus, it is possible penetrators survive because they never receive an AI assignment. If an interceptor is assigned, the penetrator must survive the attack.

Super AI Model

The super AI scenario involves a super AI which has a long-range radar detection capability acting in the role of a control center for a limited number of less capable attack AIs. The super AI patrols some portion of the grid, as dictated by its capabilities and the number of AIs available. The attack AIs are stationed at a CAP position at the center of the super AI's orbit. This center point is used for making computations for flying from the CAP and returning even though the AIs may be in cells surrounding the midpoint.

The user specifies the number of super AIs available to this type of defense and the pattern they fly. The super AIs are positioned side by side so there is no gap between their radar coverage. If the length of coverage provided by all AIs is smaller than the depth of the base case coverage zone, then the

penetrators are not liable to intercept until reaching the beginning of super AI coverage. This reduces the time the penetrators spend in coverage. If the width of the total coverage provided by the super AIs is less than the width of the base case zone, then the penetrators which enter in the uncovered areas survive since they enter outside the coverage. These penetrators are added to those surviving the coverage for comparison with the base case (see Figure 11).

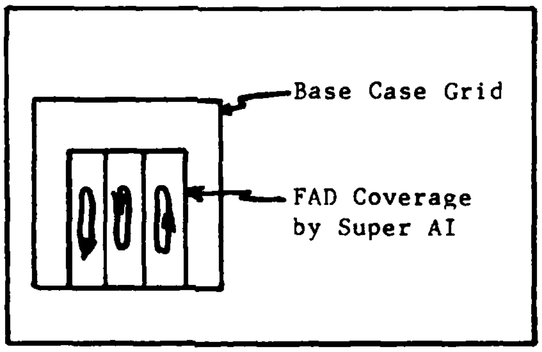

Figure 11. Super AI FAD Coverage

The depth of the rows is determined by the length of the interval Δt, which reflects how often the C^2 system can make interceptor assignments. The penetrator will advance a distance, $v_p \Delta t$, in this time. The determination of Δt relates to the time for the super AI to complete one cycle of its search pattern. Although it only sees the points at the end of its cycle once each time during its search, it sees many of the intermediate points more frequently.

A super AI does not make intercept attempts, even if a penetrator is detected and no attack AIs are available to it. The super AI is assumed to be available the entire time penetrators

are in the FAD, either through planned immediate replacement, refueling of the super AI, or extra fuel tanks. The super AI is not capable of the complete vectoring of the attack AIs to attack position and launch, but only intermittently gives directions to guide the attack AIs. Thus, the time for an attack interceptor to search and acquire is increased an appropriate amount by the user to account for the lessened degree of control possible. Since the penetrators chose an entry path at random, this model can be solved for just one of the super AIs and then multiplied by the appropriate number of super AIs to get the results for the portion of the FAD zone covered by the super AIs. Any penetrators which enter and are not detected must be added to those which survive the coverage to get the results for the FAD zone.

The probability a penetrator is detected is an input defined by the user, but the penetrator's location at the time of detection is not known. Assume that it may be detected in any one of the rows in the grid with equal probability. If it is detected in row 1, it is liable to intercept in rows 1 to r; if it is detected in row 2, it is only liable to intercept in rows 2 to r. In general, if it is detected on some row j, it is liable to intercept only in rows j to r. Assume a penetrator may be detected at any row j along column i, where j ranges from 1 to r. Once a penetrator is detected, its location is known for the rest of the time he is in coverage. Penetrators exit the FAD when they leave the last row regardless of when detection occurs.

$$P_d(i) = \frac{P_d}{r} \qquad \text{for } i = 1,2,\ldots,r \qquad (3.13)$$

and where P_d is the probability of detection as defined by the user, so that $P_d(i)$ is the probability of detection in row i.

The remainder of the calculations proceed as in the general model.

Autonomous Model

The weakest defensive situation occurs when the AIs have no control center. This could occur if no SUAWACS or super AIs are available to defend a certain corridor. In the absence of a control center, the interceptors act autonomously. They scramble and take a position in the forward air defense zone. As in the other models, only a subset of the FAD zone may actually be in the grid covered by the interceptors since there are limits on their combat radius.

The grid is divided into cells whose size is defined by the search pattern flown by the interceptors. The basic search pattern used here is a race track shown in Figure 12. The length of the cell is equal to the length of the search pattern plus twice the radar range of the AI and the depth is the depth of the search pattern plus the length of the radar.

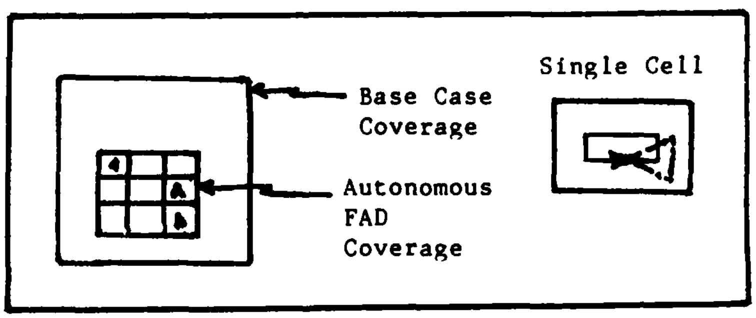

Figure 12. Autonomous FAD Coverage

Although the grid is divided into cells representing the search capabilities of the interceptors, there may not be enough AIs to place one in each cell. The AIs are distributed at random in the grid, so there may be more or less than 1 AI in a cell.

This model has a different underlying philosophy from the model used in the more advanced C^2 systems. In this model, the C^2 system is the individual interceptor. These interceptors are scattered throughout the grid, instead of being located at one central CAP. Each one must detect the penetrator before making the intercept attempt. A penetrator surviving an intercept attempt flies forward to the next cell. If there is an interceptor in this cell, this AI must make a detection and then make the attempt.

The command and control system checks the grid for penetrators each time a penetrator enters a cell where an AI may be located. In the previous cases, the length of Δt determined the width of the row since a penetrator would advance this distance in the time between C^2 checks. In length of Δt, since the penetrator encounters a C^2 system, the interceptor, each time it flies through a cell.

To find the expected number of survivors, a distribution for the probability of kill is developed. This considers three distinct phases: detections, attacks and kills. A cell may contain one, two, or more interceptors. The number of detections which are made is equal to the number of AIs in the cell which make at least one detection. Although an AI may make more than

one detection, he will attempt to intercept the first detection. The second phase deals with the number of attacks which will occur. In this situation, two or more AIs may detect the same penetrator and attempt an intercept, due to the lack of communication between AIs. Once they are within visual range, by mutual agreement, only one AI will make the attack. The third phase is the resulting number of kills which occur from each attack.

The number of detections which can be made is binomially distributed with parameter n and p, where n is the number of AIs and p is the probability of detecting at least one penetrator. The number of kills is also binomial with parameter n, the number of attacks, and p, the probability of kill. The number of attacks made out of j detections is the cell occupancy problem derived in Appendix D. Essentially the number of attacks which can be made is equal to the number of different penetrators detected. Consider the penetrators as cells and the interceptors as balls, then the problem is to find the number of occupied cells.

The general form of the distribution of X, the number of killed penetrators is:

$$P(X=i) = \sum_{j=i}^{NAI} \left[\binom{NAI}{j} [1-(1-P_d)^{NP}]^j [(1-P_d)^{NAI-j} \left\{ \sum_s \binom{s}{i} P_k^i (1-P_k)^{s-i} \right. \right.$$

$$\left. \left. P_{NP-s}(j,NP) \right\} \right] \quad (3.14)$$

where

NP - total number of penetrators

NAI - total number of interceptors

i - the number of kills $i = 0,1,\ldots,\min(NP,NAI)$

j - the number of detections $j=(i,i+1,\ldots,NAI)$

P_d - the probability of detecting a single penetrator

s - the number of unique detections

$$\text{lower limit} = \begin{cases} 1 & \text{if } i=0,1 \\ i & \text{o.w.} \end{cases} \quad \text{upper limits} = \min(j,NP)$$

P_k - the probability of kill

$P_{NP-s}(j,NP)$ - the probability of (NP-s) attacks when j detections of NP penetrators are made.

$$P_{NP-s}(j,NP) = \binom{NP}{NP-s} \sum_{v=0}^{s} (-1)^v \binom{s}{v} \left(1-\frac{(NP-s)+v}{NP}\right)^j \qquad (3.15)$$

m =(NP-S) empty cells; penetrators with no AI assigned

r = i of NAI interceptors which make detections

n = NP penetrators

The complete derivation of this distribution is given in Appendix C.

The expected number of kills can then be calculated from this distribution and subtracted from the number of penetrators. This becomes the number of penetrators surviving which enter the next cell and must be detected by the interceptors occupying this cell before the intercept attempt can be made. The number surviving which exit the FAD from the last row at each interval is summed to determine the total number of penetrators surviving.

In this model, the number of interceptors available for assignment is accounted for on a cell-by-cell basis, since the AIs act autonomously. The AIs in the cell make the detection and then assign themselves to make the interceptor. There is no larger

C^2 system which makes assignments. The actions in one cell do not influence the AI availability in another cell. More than one AI may occupy a cell, but no communication occurs between the two providing information on the detection of penetrators.

The AIs take some average time to make an intercept attempt since they do not have to fly from a CAP position to the penetrator, but are located in the same cell with the penetrator. With more advanced interceptors and increasing cell size, the flying time can be calculated. Since the AIs act autonomously, the data processing time, T_A, and the time for the AI to search and acquire, T_S, are taken together as one time representative of the longer processing time required by the less capable C^2 system. Assume the individual AIs will use either 1 or 2 shots. If the AI uses only 1 shot, it returns to searching for penetrators. If it has used both shots, the AI must return to base and rearm. Another AI is sent to replace this AI.

The number of AIs is reduced by the expected number of interceptors which make detections for the length of the intercept. Since not all detections will result in an attack where the AI uses up its weapons, some AIs return to searching for penetrators with both shots. As before, the equivalent number of two shot AIs return to the cell, representing AIs with 1 shot left. The AIs which use up both shots are replaced by another AI at a later time period.

Computerization and Verification

The three models were coded in FORTRAN IV. The outputs for

the close control cases are the expected number of penetrators undetected, the expected number of penetrators never assigned, the expected number of penetrators surviving all attacks and the expected number of penetrators surviving the FAD. In the autonomous model, these subdivisions can't be made and only the number surviving the FAD is given. The user can insert additional print statements to look at intermediate results of the models or the interceptor availability.

These models were verified in three phases. The first phase involved building and then verifying the model in small pieces. These included the sections on flying time, interceptors availability as a function of time, and the expected number of penetrators surviving calculations. Also, for the autonomous case, the occupied cell problem was first verified, then the kill probabilities were added and verified and finally the detection probabilities added and verified.

After all the pieces of the models were integrated, an example problem was selected for each model, solved by hand and compared with the computer results. The results agreed in all cases. IF/THEN/ELSE blocks which were not tested with these examples had a modification made to the example which was then used to check these blocks. The portion of the results was reworked in accord with the modification of the example. These results also agreed.

A third phase of the validation effort which is considered in this section rather than in the results of Chapter V, is the sensitivity of those parameters which are hard to determine.

Specifically, the C^2 cycle time and the number of rows in the grid. This analysis used the base case described in Chapter V. All other inputs remained fixed, and the C^2 cycle time was varied. The C^2 cycle time determines the number of rows and the number of time intervals the penetration lasts.

Table 2 shows parameters and the ranges which were tested. In the SUAWACS and super AI case, the number of rows is determined by the C^2 cycle time. In the autonomous case the cell represents the area a single AI covers. The size of the cell relative to the grid determines how many cells fit in the grid and the actual coverage provided. For this analysis, a basic search pattern with a length of 10 miles and depth of 4 miles was assumed. The radar ranges considered were: 10 miles, 15 miles, 20 miles, and 45 miles. This produces cells of length and width: (30x24), (40x34), (50x44), and (55x49). The coverage zones were: (510x504), (520x510), (500x484), and (500x470). Because of the numerous ways to specify a search pattern and radar range only these few were chosen for examination.

The range of C^2 cycle time is fairly tight for the SUAWACS case since its specification is relative to the other models. The results for scenarios 1, 2 and 3 did not significantly change for a cycle time of 1,2,3, or 4 minutes. Scenario 4 results changed when a cycle time of 4 minutes was reached.

The range considered for the C^2 cycle time of the super AI is much larger than that of the SUAWACS since its determination is dependent on capabilities and tactics. These changes in the cycle time do effect the expected number of penetrators surviving, as

TABLE 2

C^2 Input Sensitivities

SUAWACS					
C^2 Cycle Time	1 min	2 min	3 min	4 min	5 min
Number of Rows	62	31	21	16	13
Length of Penetration (In Intervals)	91	45	30	23	18
Super AI					
C^2 Cycle Time	3 min		8 min	10 min	15 min
Number of Rows	21		8	6	4
Length of Penetration (In Intervals)	30		11	8	5
AUTONOMOUS					
Radar Range	10 miles		15 miles	20 miles	45 miles
C^2Cycle Time	2.9 min		4 min	5.68 min	
Number of Rows	21		15	11	5
Number of Columns	17		13	10	5
Length	31		22	16	7

illustrated in Figure 13.

The range of the cell size for the autonomous case varies from a cell size of 30x24 to 55x49. The area covered in the latter cell is 3.7 times the area covered in the first. The results are shown in Figure 13. In the case of a small penetrating force, the larger radar coverage is better. However, as the penetrator size increases the smaller radar coverage provides better results. This is because there are more AIs in any cell when there is a larger radar coverage. As a result more AIs make detections on the same penetrator and there are fewer attacks and kills as a result.

Summary

The models which have been developed are based on a single underlying set of concepts which can be applied to the desired FAD scenario. This involves structuring the forward air defense zone into a grid representing the coverage provided by the C^2 system, and allows the model to track the penetrators as they fly through the grid. As a penetrator passes through each row an intercept attempt can be made on it if AI resources are available. The size of each row reflects the command and control capabilities of the system being modeled. Some length of time Δt, is required by the C^2 system to cycle and check the entire grid for penetrators. Since the velocity of the penetrators is known, the distance covered in this time Δt, can be calculated giving the row depth. At each interval Δt, or equivalently, as a penetrator passes through a row, intercept attempts are made against the

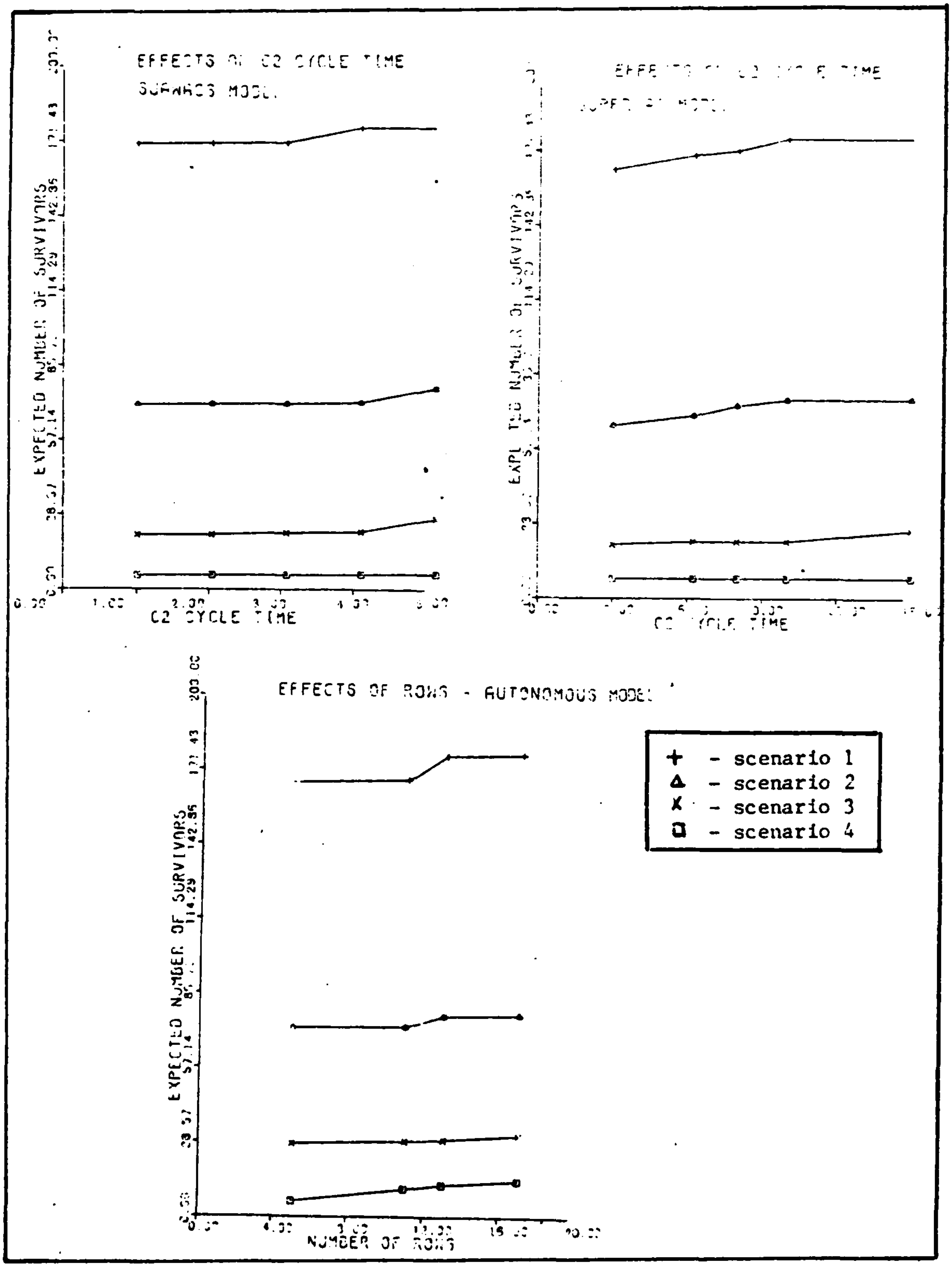

Figure 13. Effects of Rows

penetrators. The number of penetrators in each row is modified to reflect the expected number of kills. These become the survivors which enter the next row at the following time interval. The number of interceptors available is modified to reflect the number of AIs involved in intercepts and the time required to replace them. Tables 3 and 4 provide a brief summary of the assumptions of the basic model and assumptions for C^2 aspects of each scenario.

TABLE 3

Model Assumptions

Assumptions--Base Model	Rationale
Penetrators	
1. Penetrator force homogeneous	Simplicity
2. Penetrators do not fire on AIs.	Simplicity
3. Penetrators travel at a constant speed.	Simplicity
4. Attrition due to AIs only.	Scenario representation, only considering effectiveness of FAD
5. Entry point of penetrators chosen at random.	Defense has no knowledge of offensive tactics.
6. Penetrators fly in straight lines parallel to sides of grid.	Simplicity
7. Penetrators fly at a constant altitude.	Simplicity
8. Equal priority of penetrators needing assignment.	Simplicity
9. Entry window for penetrator arrival	All penetrators try to arrive within same space of time to saturate defenses.
10. Penetrator located in center of cell at each time interval	Ease of calculating time to fly to penetrate.

TABLE 3

Model Assumptions

Assumptions--Base Model	Rationale
Interceptor	
1. Interceptor force homogeneous	Simplicity
Equal priority of penetrators needing assignment.	Simplicity
Entry point of penetrators chosen at random.	Defense has no knowledge of offensive tactics.
Penetrators fly in straight lines parallel to sides of grid.	Simplicity
Entry window for penetrator arrival.	All penetrators try to arrive within same space of time to saturate defenses.
Penetrator located in center of cell at each time interval	Ease of calculating time to fly to penetrate.
2. Two identical shots	Address reality of situation.
3. Shoot-look-shoot capability	AIs will be able to determine success/failure of first shot before firing the second; within visual range.
4. No additional time required for AI to make second shot.	Time to convert and fire a second time negligible relative to other times involved.
5. AIs replaced after both shots are fired with appropriate delay	AIs must be replaced not an infinite supply of weapons, and there is a delay associated with replacement.

TABLE 3

Model Assumptions

Assumptions--Base Model	Rationale
Interceptor (Continued)	
6. Constant number of AIs committed to air defense.	Simplicity
7. AI availability due to not modeled fuel.	AIs can remain airborne for length of penetration.

TABLE 4

Assumptions Due to C^2 Aspects

Assumptions - C^2 Aspects	Rationale
1. No simultaneous intercepts, only one-on-one engagement	Simplicity in autonomous case. In close control reflects assignment policy.
Close Control	
1. Probability of detection a function of C^2 system only.	
SUAWACS: Occurs in first row	Reflects superior radar capability
Super AI: Occurs at random in each row.	Reflects the unknown location of super AI when penetrator enters.
2. Once a penetrator is detected, he remains detected.	Advantage of control, ability to generate tracks.
3. Perfect vectoring of AI to penetrator by C^2 system.	Reflect Soviet C^2 philosophy of involvement until end of intercept.
Autonomous	
1. Probability of detection a function of the individual AIs.	Knowledge of penetrators not retained, must be reacquired.
2. No communication between AIs.	Each AI acts on the first detection it makes.
3. AIs are placed at random on grid initially.	AIs scramble and are only given a general zone to provide coverage. Each chooses a position at random.

CHAPTER IV

APPLICATION OF THE MODEL

This chapter is designed to provide the user with some suggestions on methods of deriving input values. It also has a small-scale application which may further the user's understanding of the model.

Developing Model Inputs

This model has many parameters which are specified by the user. This provides a large amount of flexibility in the application of the model. The limited sensitivity analysis presented at the end of Chapter III indicates to some degree, the sensitivity of the C^2 cycle parameter.

This chapter discusses the inputs the user must provide and some suggestions for their derivation. An example is provided to illustrate the selection of inputs and the model results. Table 5 is a list of the inputs the user provides.

Grid Size. The model considers the effectiveness of the defense on an area of length L and width W. This area may be bounded by either the maximum coverage of the defense or the penetration corridor. In this example, the grid size is:

Grid Size	
Length	50 miles
Width	30 miles

TABLE 5

MODEL INPUTS

Grid Size

- Length
- Width

Penetrator Inputs

- Number of Penetrators
- Entry Window
- Velocity

Interceptor Inputs

- Velocity (dash)
- P_k
- Time to replace
- Number of interceptors

Detection Probability (P_d)

- Close control
  - SUAWACS
  - Super AI
- Autonomous

Control Capabilities

- SUAWACS
- Super AI

C^2 Considerations

- SUAWACS
  - Cycle Time (Δt)
  - Number of rows
  - Number of columns
- Super AI
  - Cycle time (Δt)
  - Number of rows
  - Number of columns
  - Number of Super AIs
- Autonomous
  - Cell size
  - Number of rows
  - Number of columns
- Times for Intercept
  - SUAWACS-Super AI-Autonomous
  - T_A Data processing time.
  - T_S Time for individual AI to search and acquire when in vicinity.
  - T_L Time to convert and launch.

Penetrator Inputs. Penetrator inputs reflect the type of threat the defense expects. The scenario dictates the size of the force and their entry strategy. Many penetrators may represent a cruise missile force, a small number may be a bomber force. The penetrators may enter over the same time period to try and saturate the defense. The entry window is the time between first and last penetrator entries to the FAD coverge zone. For this example, define:

Penetrators:

Number of penetrators	6
Velocity	8 mi/min
Entry window	6 minutes

Interceptor Inputs. The interceptor inputs reflect some of the capabilities of the defense. The interceptor's dash velocity is the high rate of speed that the AI uses when flying to make an intercept attempt. This input is necessary for the close control models since they are flying from a CAP position to the penetrator. In the autonomous case, the penetrator and AI are collocated. In this case the AI velocity while searching may impact on the probability of detection.

Replacement time may be used to indicate the location of the support bases relative to the FAD coverage zone.

The kill probability the user provides is the single shot kill probability of a single "shot." A shot may use one or more weapons. If a shot consists of two weapons, then the P_K reflects this. There may be in-depth research used to find a kill probability for the AAM used by this type interceptor against this

type of penetrator; or it may simply be estimated relative to the entire group of AAMs available. For this example:

Interceptors

P_k (1 shot of 1 weapon)	.8
Replacement time	30 minutes
Dash velocity	15 mi/min
Number of interceptors	6

<u>Detection Probability</u>. The detection probability for the SUAWACS represents his capability to detect penetrators of a certain type. Determining the P_d for an interceptor which is searching an area, as the super AI and autonomous interceptors do, is more difficult. One suggestion offered is to have the interceptor fly in a racetrack-type search pattern, then knowing the radar capability against this penetrator and the speeds of both interceptor and penetrator, the probability an AI detects a penetrator flying through his coverage can be found. Another possible approach is to use a search theory algorithm which specifies P_d as a function of grid size, AI velocity, penetrator velocity, sweep width, etc. (Ref 21). It is important to remember that P_d for the autonomous case applies to the individual AIs, whereas P_d in the close control situation only applies to the C^2 system. In this example, a fairly representative number is chosen:

Detection Probability

Close Control

SUAWACS	1.0
Super AI	.8

Autonomous	.5

Control Capability. The control capability parameter only applies to the SUAWACS and super AI. It is a measure of the degree of control. This represents the number of intercepts the C^2 system. can handle simultaneously, it may be limited by communication channels, the number of operators, or radar tracking capabilities. If the operator vectoring the AIs can direct two interceptors penetrators by flip-flopping between the two, then he is essentially directing two interceptors. For this example,

Control channels

SUAWACS:	4
Super AI:	2

C^2 Parameters. The final set of parameters describing the C^2 capability may be the hardest for the user to accurately provide. Consider first the autonomous case. In this case, the user choses a method to divide the grid into rows and columns to reflect use of the AIs. For this example, the grid is divided into rows and columns, where each cell represents the coverage provided by an individual interceptor. The determination of cell size for this problem is shown below in Figure 14.

Assume the interceptor's radar has a 45° sweep angle and a range of 15 miles, then the radar range depth as shown in Figure 14 is 30 miles.

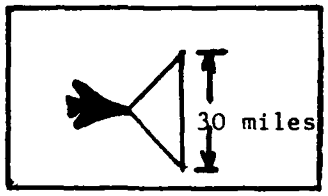

Figure 14. Radar Capabilities of Autonomous AI

Now, placing the AI in a racetrack detection search pattern determines the cell size he can cover with a specified probability of detection, P_d=.5 (See Figure 15).

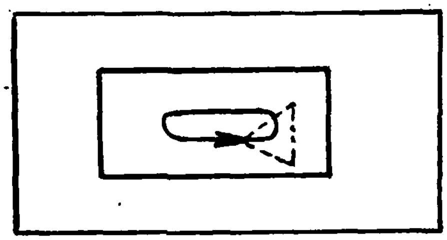

Figure 15. Search Pattern of Autonomous AI

The cell size has a width of 40 miles and a depth of 34 miles. The grid is 30 miles wide, so it is one cell wide. The grid is 50 miles long, so it will hold 1.4 cells. If some other search pattern is used by the AI, then the area covered in the search must be translated into an equivalent area which has probability of detection P_d=.5, contained in a rectangle. This rectangle becomes the cell size used to divide the grid.

The grid size in the autonomous case represents an area the defense attempts to cover. The AIs are distributed at random in this area. Thus, the grid may not divide into an even number of cells. For instance, if the grid length was 90 miles and the cell size remains (40 x 34), then the grid could be overfit or underfit

as shown in Figure 16. The suggestion is made that if that remaining length of the grid which is not covered is less than one-half the row length, then the grid be underfit in rows; if it is greater, then the grid be overfit. Similarly, the remaining width of the grid which is not covered should

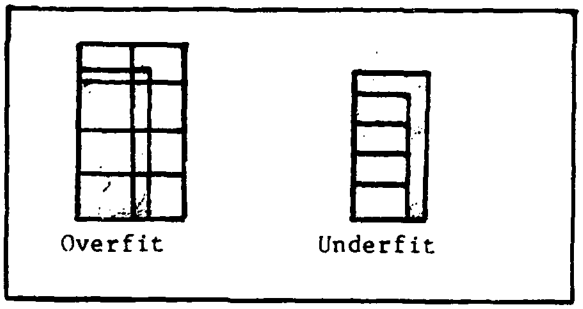

Figure 16. Overfit and Underfit Grids

be underfit or overfit in columns. This is only a suggested method for handling this problem--the user may simply want to drop the uncovered portion of the grid in all cases since this would yield a conservative estimate on the effectiveness of the defense. The user should be consistent in applying the same rule for all cases.

In the autonomous case, a penetrator may encounter an AI each time it enters a new row. Thus, it may be detected and have an intercept attempt made on it in each row. In this example the grid is underfit and only contains one row. All the interceptors are located in this row. The velocity of the penetrator is 8 mi/min, the row is 34 miles deep so the penetrator flies through a row every 4 1/2 minutes.

C^2 Capabilities--Autonomous

Cycle time 4 1/2 Minutes

Number of rows 1

Number of columns 1

In the super AI case, the C^2 cycle time reflects how often the AI checks his portion of the entire grid. The AI can be represented using its radar range and a racetrack pattern. Suppose this super AI has a radar range of 20 miles and a sweep angle of 45^{o} (see Figure 17).

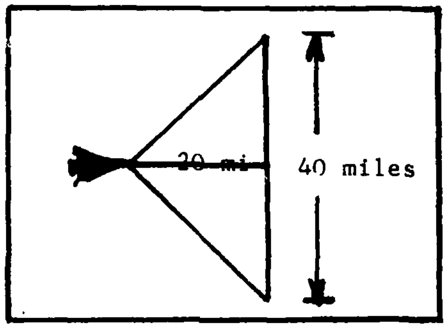

Figure 17. Super AI Radar Coverage

Suppose the super AI flies the racetrack pattern shown in Figure 18. The maximum area the super AI could provide is indicated.

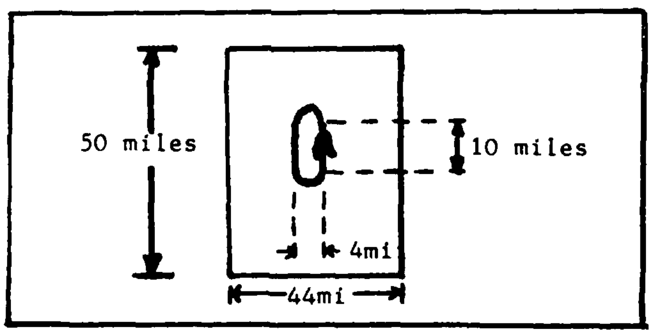

Figure 18. Maximum Coverage Area of Super AI

Since the depth of the grid is 50 miles, then the super AI sees points such as A and B once each time he completes a search pattern. If he flies at a loiter velocity of 6 miles/minute, then

he will complete one search pattern every 4 2/3 minutes. The super AI will have point C in coverage 1 2/3 minutes every search pattern. In Figure 19 the legs of the pattern when point C is in coverage are solid. The dashed lines show when it is not in coverage.

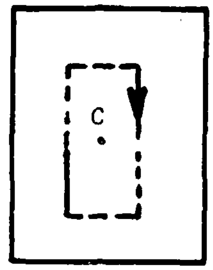

Figure 19. Super AI Coverage of a Point

A simple average these two values will provide a C^2 cycle time of 2.8 minutes. A more complicated weighted average of a series of pts between A and C may be used. Or the user may elect to use a more sophisticated method for finding recycle time. If a different search method is used, the area covered can be converted to an equivalent rectangular area to find the coverage provided by a single AI for grid size determination.

In this example, super AI is sufficient to cover the zone being considered. The user may place super AIs side by side on the grid until it is full. The depth of the super AI coverage may not cover the grid. The effect on the defense is a smaller period of time that the penetrator is in the coverage zone and thus liable to attack. The width of the coverage may also be smaller than the grid. The number of penetrators entering through this width survive penetration of the grid since they are outside AI coverage.

A C^2 cycle time of 2.8 minutes allows the penetrator to advance 22.4 miles between C^2 checks. This is the row depth. For a grid size of 50 miles, 2.23 runs of 22.4 miles will cover the grid. Using the rule of underfitting the grid, the super AI coverage will have two rows. Columns are used to provide a reference for the computation of the time to fly to the penetrator. To keep this example simple, only one column is defined.

C^2 Aspects - Super AI	
C^2 Cycle time	2.8 minutes
Number of rows	2
Number of columns	1

The SUAWACS checks the grid continuously; it is superior to the super AI and autonomous model in this respect and should have a better C^2 cycle time. In this example, a C^2 cycle time of two minutes will be used. In two minutes the penetrator advances sixteen miles which is the depth of a row. The grid can be divided into 3.125 rows; in this instance, the grid will be underfit using three rows. The number of columns used to compute flying time is one.

C^2 Aspects - SUAWACS	
C^2 Cycle time	2 minutes
Number of rows	3
Number of columns	1

The user may wish to make the C^2 cycle times divide the grid's length and width into an integer number of rows and columns. If this is done, accuracy in the C^2 cycle time is

traded to make the actual coverage equal the grid size. This might be desirable in comparing across models. In the examination of a single model, an accurate determination of the C^2 time, and thus the actual coverage zone would seem to be a more important consideration.

Times to Make the Intercept. These inputs represent times for data processing, search and acquisition by the individual AI, and time to convert and launch the missile. The time for data processing represents the time for the C^2 system to verify the existence of a penetrator and decide on a course of action. This will probably be smaller for the SUAWACS than for the super AI. The time for the individual AI which is vectored to the near vicinity of the penetrator to acquire the penetrator on its radar, is a function of both the capability of the C^2 system to accurately vector the AI to the penetrator and the radar capabilities of the AI. The more capable AIs will require less time, and a more capable C^2 system requires less time. In the autonomous case, these two actions are interleaved and one time can be used to represent both actions. The time to convert and launch a missile is a function of the interceptors and missiles used which will probably be standard across all models.

Time to Make the Intercept

	SUAWACS	SUPER AI	AUTONOMOUS
T_A	1.0	1.5	4.5
T_S	1.5	2.0	
T_L	.5	.5	.5
	3 minutes	4 minutes	5 minutes

Summary. These are only suggestions for determining the value of the input parameters. Better results are produced with more accurate parameter inputs. The sophistication the user applies in developing the input values should be guided by the intended use of the model results. As time and resources permits, the inputs may range from good "guesses," to outputs from other models addressing these aspects, to special user written algorithms. This section is intended to clarify input determination and provide some suggestions. The amount of effort and sophistication which goes into determination of the inputs should be commensurate with the intended use of the model and the time available.

Example

The input values for each of the following problems are given in the previous subsection. Briefly, for a 30 x 50 grid, the effectiveness of each of the defenses manned with 6 AIs against 6 penetrators is examined. For each model, results are calculated.

SUAWACS. Since the C^2 cycle time is two minutes, two penetrators will enter each time interval, and three time

intervals are needed for all the penetrators to enter the grid. It takes two more intervals for the penetrator to fly through the grid, so the last of the penetrators exit after time interval 5.

For brevity in presenting the results, the following notation will be used:

NEED AI - average number of penetrators requiring interceptor assignment

C^2 AVBL - average number of C^2 control channels available

AI AVBL - average number of AIs available

AI ASSIGN - average number of penetrators which have never had an AI assignment

TOTAL C^2 BUSY - average number of C^2 channels occuppied at this interval

TOTAL AI ASSIGN - average number of AI which have been assigned and are unavailable at this interval

AI ATTACKS - In the autonomous case this refers to the average number of AIs which actually make the attack against the penetrators. Limited to the number of different penetrators detected.

*SURV - average number of penetrators which have survived one or more AI attacks

1 SHOT AI - average number of interceptors with one shot left

*EQUIV AI - the equivalent number of AIs which are returned representing those with one shot left

*0 SHOT AI - average number of AIs which will be replaced by another AI

A value in parentheses will follow each of the values of the (*)

variables. For SURV, this is the row number where the penetrators are located after the intercept attempt against them has failed and they again need an assignment. If it is greater than the number of rows in the grid, then the penetrators are outside of coverage, will not be reattacked and thus survive the FAD coverage. For EQUIV AI and 0 SHOT AI this is the time interval when these AIs return or are replaced at CAP. If it is greater than the length of the penetration, then these AIs will obviously not be available to attack penetrators since they would return after all penetrators have exited the grid. For C^2 AVBL it represents the time interval when the channels become free, (i.e. when the intercept attempt is complete).

Figures 20 and 21 show the average number of penetrators surviving which are detected by the C^2 system and need assignment. Figure 20 shows the surviving penetrators and their location in the grid at the beginning of each interval. Figure 21 shows the results of the effect of the AI assignment for the end interval. Table 6 summarizes the events.

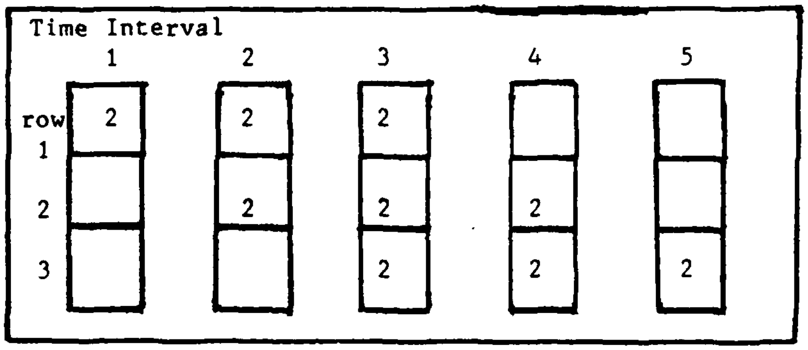

Figure 20. SUAWACS Grid at Interval Start

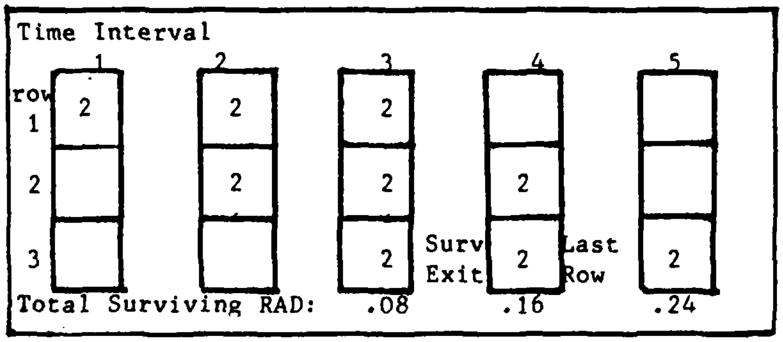

Figure 21. SUAWACS Grid Interval End

TABLE 6

SUAWACS EXAMPLE RESULTS

Time Interval	1	2	3	4	5
At Beginning of Interval					
NEED AI	2	2	2	2	2
C^2 AVBL	4	2	0	2	2
AI AVBL	6	4	2	2	0
C^2 ASSIGN	2(4)	2(5)	0	2(7)	0
AI ASSIGN	2	2	0	2	0
TOTAL C^2 BUSY	2	4	4	4	2
TOTAL AI ASSIGN	2	4	4	4	2
At End of Interval					
UNASSIGN	0	0	2	0	0
SURV	.08(4)	.08(4)	0	.08	0
1 SHOT AI	1.6	1.6	0	1.6	0
EQUIVAL	.8(7)	.8(8)	0	.8(9)	0
0 SHOT AI	.4(11)	.4(12)	0	.4(15)	0
TOTAL SURVIVING FAD			.08	.16	.24

In time interval one, two penetrators enter and both are detected by the SUAWACS. Since AIs and control channels are available, two AIs are assigned to these penetrators. The time required for the intercept in this case, is longer than the C^2 check time. So at time interval two, these penetrators are not

considered with the other penetrators needing assignment since an intercept is in progress against them. The resulr of the intercept is not known until some later time interval. At this later interval, the surviving penetrators could again be reassigned if they are still in the FAD coverage zone. In this example, the .08 penetrators surviving the attack have exited the coverage area and are not liable to reassignment. This number of survivors is added to the total surviving the FAD at the appropriate time. In Figures 20 and 21 this is indicated by circling the penetrators which require an interceptors assignment each interval. Numbers which are uncircled represent the penetrators under attack, which do not require assignment since the intercept attempt is still going on. When they exit the grid, the survivors of this interval are added to the total surviving the FAD. In this example, .08 of the 2 penetrators attacked survive.

This also occurs against the penetrators which enter at time interval two. In the third time interval, two penetrators enter and are detected but the control channels are all busy, so no AIs are assigned. At the fourth time interval, the intercept against the penetrators which entered during the first interval is complete and two control channels are available to make an assignment against the two penetrators in row 2, which needed assignment. In the fifth interval there are intercepts continuing against all penetrators in the grid, so no assignments are made at this time.

Eighty percent of AIs which are assigned at any interval use

only I shot in the intercept attempt, so an equivalent number of two shot AIs are returned to CAP. For this example, 1.6 of the 2 AIs assigned in the first interval use only one shot, so .8 equivalent AIs are returned to the CAP. The remaining twenty percent, or .4 use both shots and are replaced by another AI.

Super AI. This illustrates the super AI defense operation. The defense only acts against those penetrators it detects. The cycle time for the super AI is 2.8 minutes. It takes three intervals for all the penetrators to enter the grid, and it takes more interval to fly through the grid, so the last penetrators would exit after time interval 3. Table 7 shows the results of each time interval. Figures 22 and 23 show the location of the penetrators at the beginning and end of the intervals.

TABLE 7

SUPER AI EXAMPLE RESULTS

Time Interval	1	2	3	4
At Beginning of Interval				
NEED AI	2.24	2.48	2.56	.32
C^2 AVBL	2(3)	0	0	2
AI AVBL	6	4	4	4
C^2 ASSIGN	2(4)	0	0	.32
AI ASSIGN	2	0	0	.32
TOTAL C^2 BUSY	2	2	2	2
TOTAL AI ASSIGN	2	2	2	2
At End of Interval				
UNASSIGN	.24	2.48	2.56	.32
SURV	.08(3)	-	-	.013
1 SHOT AI	1.6	-	-	.256
EQUIV AI	.8(4)	-	-	.128
0 SHOT AI	.2(11)	-	-	.064
TOTAL SURVIVING FAD		.32	2.56	2.573
NUMBER UNDETECTED	1.2			
TOTAL SURVIVORS	4.373			

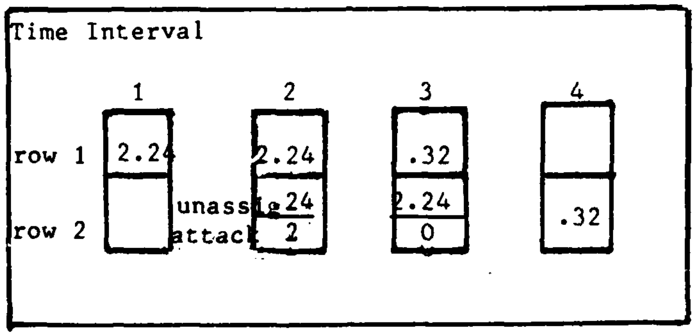

Figure 22. Super AI Grid at Interval Start

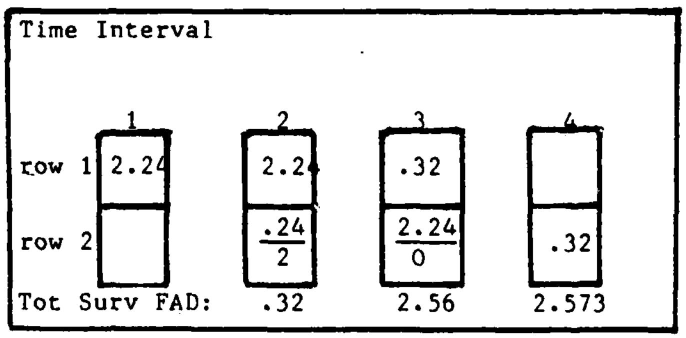

Figure 23.. Super AI Grid at Interval End

In the super AI example, the probability of detection is less than one, so 1.2 of the 6 penetrators survive because they are undetected. The remaining penetrators must survive attacks made on them as they pass through the FAD coverage region. In the first interval 2.8 penetrators enter, and 2.24 are detected. There are 6 AIs available, but the super AI only has 2 control channels, so only 2 AIs are assigned. This leaves .24 penetrators unassigned, which will be eligible for assignment at the next time interval. The penetrators which survive the attack are not liable to assignment until the intercept is complete. In the example, more than one interval is required.

In the second interval, since all the control channels (2) are busy, none of the 2.48 penetrators receive an AI assignment. Of the 2.48 penetrators needing assignment, 2.24 are located in row1 and have just entered, and .24 are penetrators which did not receive an AI assignment in the first interval. At the end of the second interval, the penetrators in the last row, those which entered in the first interval, exit the FAD coverage region and are no longer eligible for an assignment. Those penetrators which never received an assignment survive (.24), as well as those which survived the attack made against them, (.08) and exit at this period.

In the third interval, the remainder of the penetrators enter. The control channels are still occuppied, so no intercepts will be made against any of the penetrators the super AI detects. At the end of this interval, the 2.24 penetrators in the second row (those which entered in the second time interval) will exit

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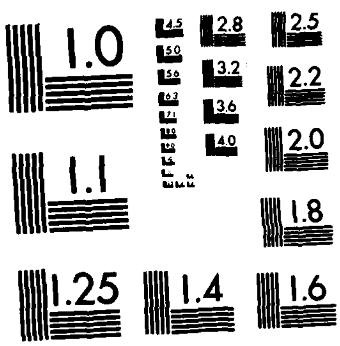

MICROCOPY RESOLUTION TEST CHART
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the FAD coverage. All of these penetrators survived because the C^2 system was unable to make assignments against them due to limited control channel availability.

In the fourth time interval, the control channels become available to vector AIs, but only a small number of penetrators are in the coverage region requiring assignment, assignments are made against these AIs, and the survivors of the attack will exit coverage before additional assignments can be made.

Autonomous. In this model the C^2 cycle time is 3 minutes, since a row is 24 miles deep and it takes the penetrator 3 minutes to fly through a row. It takes 2 intervals for all penetrators to enter the grid. There are 2 rows for the penetrator to fly through, so after entry only 1 more row is left until he exits. Thus the last penetrator who entered at interval 2 exits after interval 3. Table 8 gives the results by time interval for this defense. Figures 23 and 24 show the location of the penetrators and number in the grid for each time interval.

TABLE 8

AUTONOMOUS

Time of Interval	1	2
At Beginning of Interval		
NEED AI	3	3
AI AVBL	3	1.012
AI ASSIGN (DRT)	1.988	.882
AI ATTACKS	1.833	.880
At End of Interval		
SURV	1.24	2.16
1 SHOT AI	1.467	.704
EQUIV AI	.733	.352
0 SHOT AI	.366	.176
TOTAL SURVIVING FAD	1.24	3.4

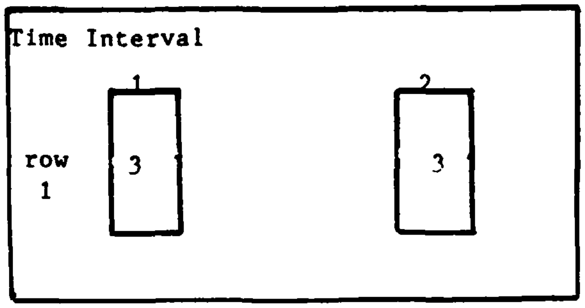

Figure 24. Autonomous Grid at Interval Start

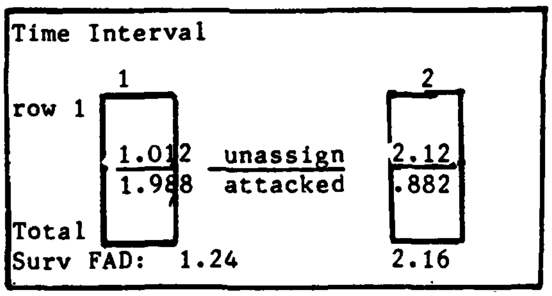

Figure 25. Autonomous Grid at Interval End

In the autonomous case three penetrators enter the first cell and encounter three interceptors. The expected number of detections made is 1.988, so 1.988 AIs begin an intercept attempt. Of these, an average 1.833 attacks will be made, with .155 AIs being waved off and returning to search without using any shots. Of the 1.833 which do attack penetrators, 1.467 use only 1 shot and so .733 equivalent AIs are returned.

There are 1.76 penetrators killed, so 1.24 survive: 1.012 survive because they are undetected, .07 survive the attack made against them. These penetrators exit the coverage after this cell and have no more attacks made against them.

In the second interval three more penetrators enter the FAD coverge. There are fewer interceptors available due to actions against previous penetrators. Of the 1.012 AIs available, on the average .882 make detections and .880 actually attack. There are 2.16 penetrators which survive: 2.12 survive because they are unattacked, and .035 survive the attack made against them. These penetrators now exit the grid and the penetration is over. The total number surviving the FAD is composed of those penetrators which survived both intervals, 1.24 and 2.16, for a total of 3.4.

Summary

This section is intended to clarify the use of the model and the development of inputs. The example is to provide the user with a better understanding of how the model works, and perhaps answer some questions on its application. The next chapter presents a more complex application of the model.

CHAPTER V

MODEL RESULTS

The model can now be applied to various air defense scenarios as defined by the user and the results analyzed for their implications on the effectiveness of the defense. Some base case scenarios are established which reflect the threat and certain parameters are chosen for analysis. The number of parameters which can be varied in the model, expands a thorough analysis of all aspects of the forward air defense into a full factorial design. Time limitations prevent an analysis of this depth from being undertaken along with the model development, so the analysis of the forward air defense only addressed a few parameters.

Four basic scenarios were established which changed the size of the penetration threat which arrived through the entry window. The parameters for the probability of kill, interceptor force strength, and number of control channels were considered over a range of values. Each one was varied over its range keeping everything else constant. In this manner the behavior of each parameter can be observed under different scenarios and the effect of a changing force size on the parameters can be assessed. Table 9 gives the base case definition. The four scenarios for differing force sizes were created by changing the size of the penetration threat.

TABLE 9

Grid Size: (Air Defense Region)	
Length	500 miles
Width	500 miles
Penetrator Inputs:	
Number of Penetrators	25-50-100-200
Entry Window	30 min
Velocity	8 mi/min
Interceptor Inputs:	
Velocity (Dash)	15 mi/min
P_k	.8
Time to Recycle	30 minutes
P_d:	
SUAWACS	.8
Super AI	.7
Autonomous	.5
Number of Interceptors	24
Control Capabilities:	
SUAWACS:	6 channels
Super AI	2 channels

TABLE 9

Times for Intercept:

	SUAWACS	SUPER AI	AUTONOMOUS
TA	2.0	2.5	
TS	1.0	1.5	4.5
TL	.5	.5	.5

C^2 Considerations:

SUAWACS:

Cycle Time	1 min
Number of Rows	62
Number of Columns	50

Super AI:

Cycle Time	6.25 min
Number of Rows	10
Number of Columns	17
Number of Super AIs	3

Autonomous:

Cell Size	50 x 50 miles
Number of Rows	10
Number of Columns	10
Cycle Time	6.25 min

The results of the analysis are presented in Figures 26 to 32 in a variety of ways to emphasize the information the results contain. In this section on analysis of results, scenario will refer to the penetration threat, model refers to the SUAWACS, super AI, or autonomous models, and parameters refer to P_K, number of AIs, and number of control channels.

<u>Parameter Effects by Scenario</u>

Figures 26 to 28 show the effect of one of the parameters on the three models, SUAWACS, super AI, and autonomous, across scenarios. The figures illustrate how varying this parameter affects the expected number of kills for each model.

<u>Effect of Kill Probability</u>. The kill probability is presented in Figure 26. For all scenarios and all models, an increase in kill probability decrease the expected number of survivors. In scenarios I, for high P_K, the SUAWACS and super AI models killed all penetrators except those which are not detected. In scenario two, the control capabilities also limit the number of kills. In scenarios three and four, interceptor availability becomes a limiting factor as well.

It is interesting to note that a low P_K values and super AI and SUAWACS models do much better than the autonomous model. But as the P_K attains high values, the difference between the three becomes relatively small.

<u>Effect of AI manning</u>. Increasing the number of AIs which man the FAD decreases the expected number of survivors. In Figure

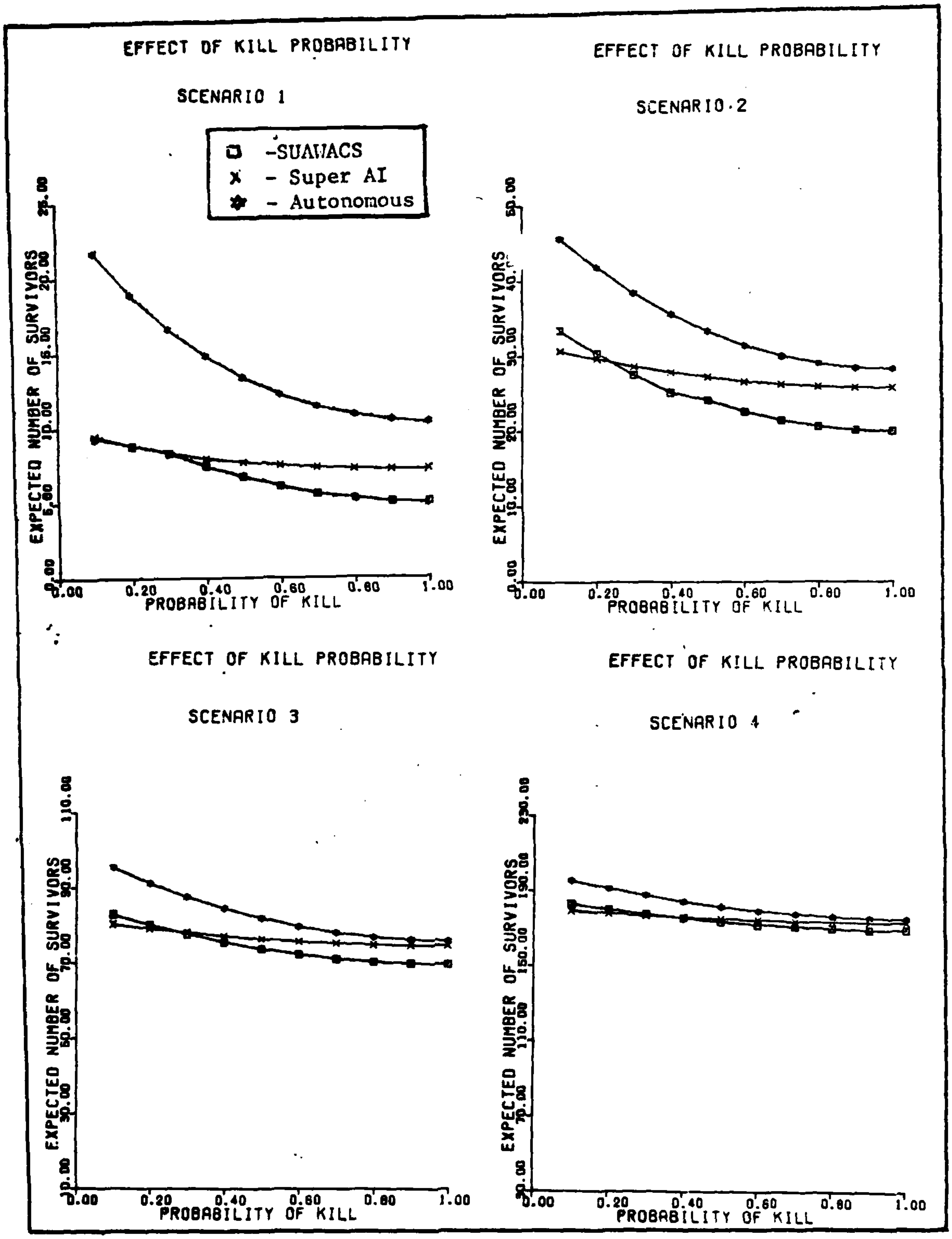

Figure 26. Effect of Kill Probability

27, the curves for the SUAWACS and super AI level off at a breakpoint. After reaching a certain level of AI manning, (approximately 20), increased numbers of AIs for the SUAWACS and super AI models do nothing towards further reducing the expected number of survivors. In the first scenario, the level of survivors achieved is chiefly composed of those penetrators undetected by the C^2 system. In later scenarios this is complicated by the limited control capability.

However, the autonomous model continues to experience reduced numbers of survivors since the C^2 system for detection and kill will depend on the quantity and quality of the interceptors. Increased AI manning increases the number of detections and kills made, and thus reduces the average number of penetrators surviving. There is no limit on control capabilities since each AI represents a control system, so none are idle for lack of a control channel. The autonomous model surpasses the other models at a level of approximately 32 AIs. At this point the ratio of control channels to AIs is one to six.

Further investigations could be done within this area to determine breakpoints as a function of the ratio of control channels to interceptors. Various combinations could be investigated to determine if changes in control channels or number of AIs provide the more profitable reduction in survivors.

Effect of Control Channels. This parameter pertains only to the close control models. A marked reduction in the number of survivors is exhibited by the SUAWACS and super AI models up until

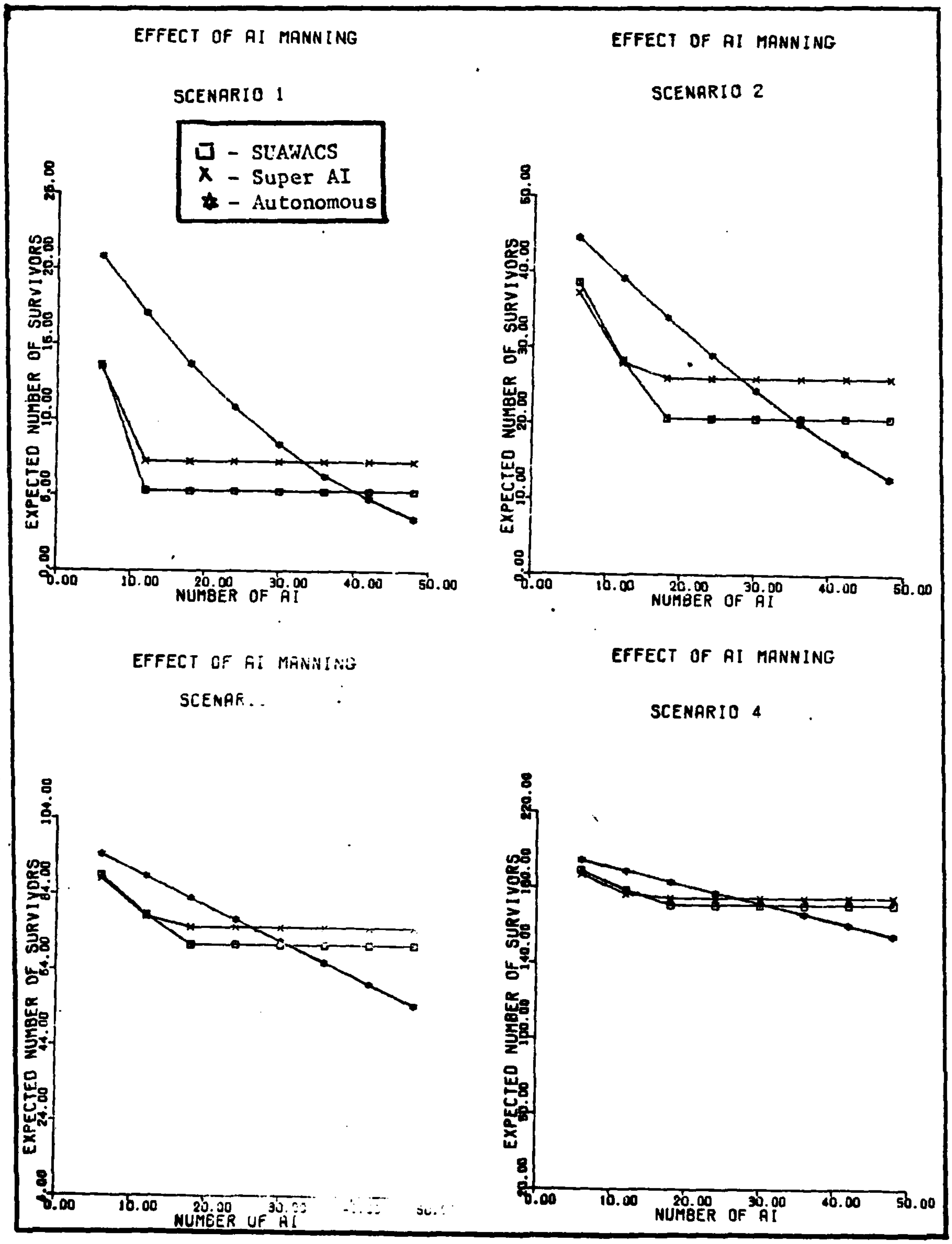

Figure 27. Effect of AI Manning

the number of control channels is equal to some breakpoint. Further improvements in control capability beyond this breakpoint does not improve the defense. This illustrated in Figure 28.

The breakpoint where increased control capability no longer improves the defense occurs at 12 control channels. This represents a ratio of one control channel to two interceptors. Increasing the control capacity above this value, leaves more channels available, but these channels must wait for AIs to return. The number of interceptors and the number of control channels are related, and the best choice for one depends on the limitations of the other. More work needs to be done to find the relationships between these two.

The size of the penetrators threat also changes the impact of improved C^2 capability. For a large penetration force, improving the control capability from 3 to 9 channels, produces a larger decrease in the expected number of survivors than the same increase in control channels against a smaller penetration force.

Parameter Effects by Model

Figures 29. 30 and 31 present the effects of one of the parameters graphed for each scenario to more clearly illustrate how the change in penetration size affects the parameter behavior. These graphs show clearly the breakpoints which occur where an improvement in the parameter's value does nothing to improve the effectiveness of the defense. In the case of the control channel parameter, the size of the penetration force, does have an impact

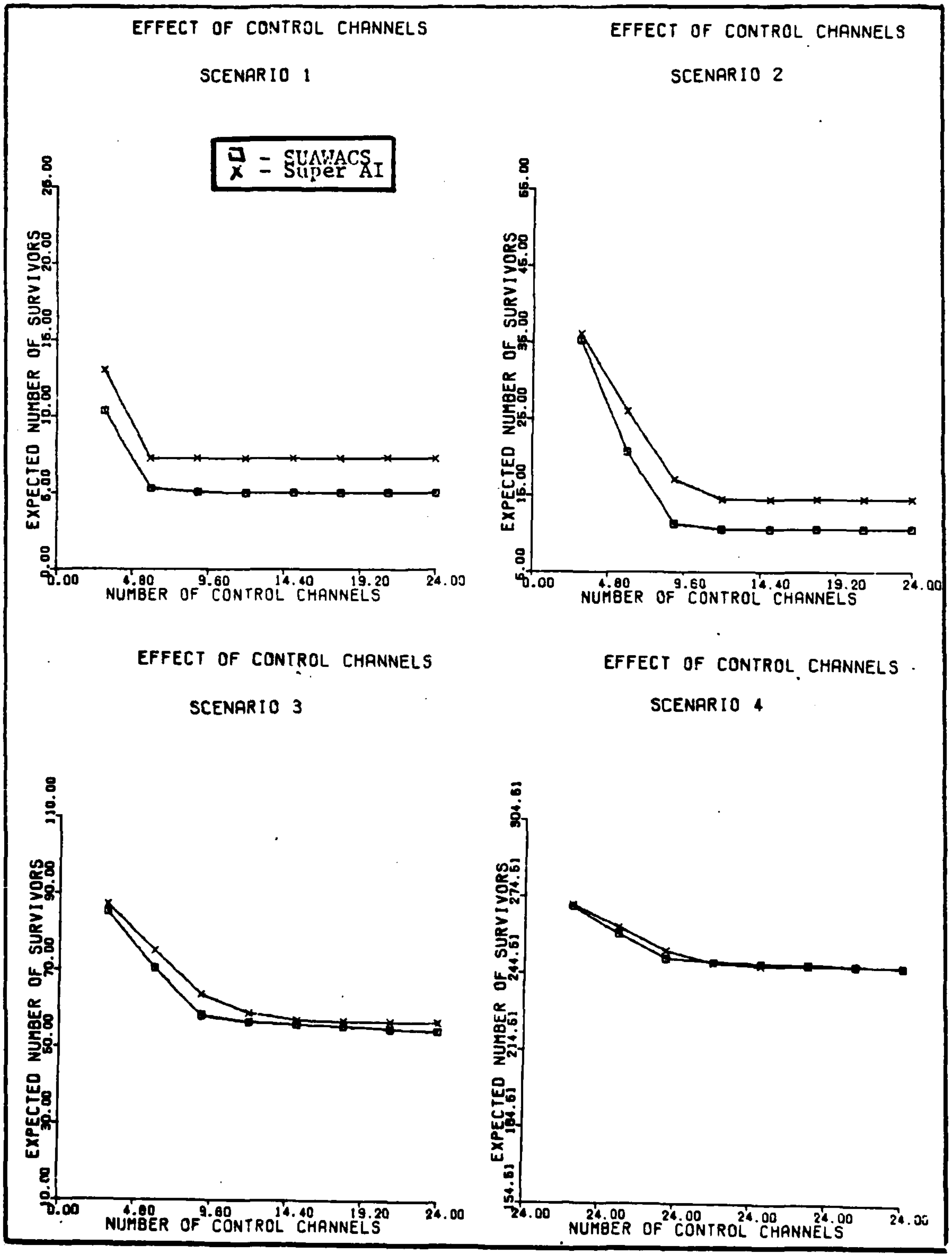

Figure 23. Effect of Control Channels

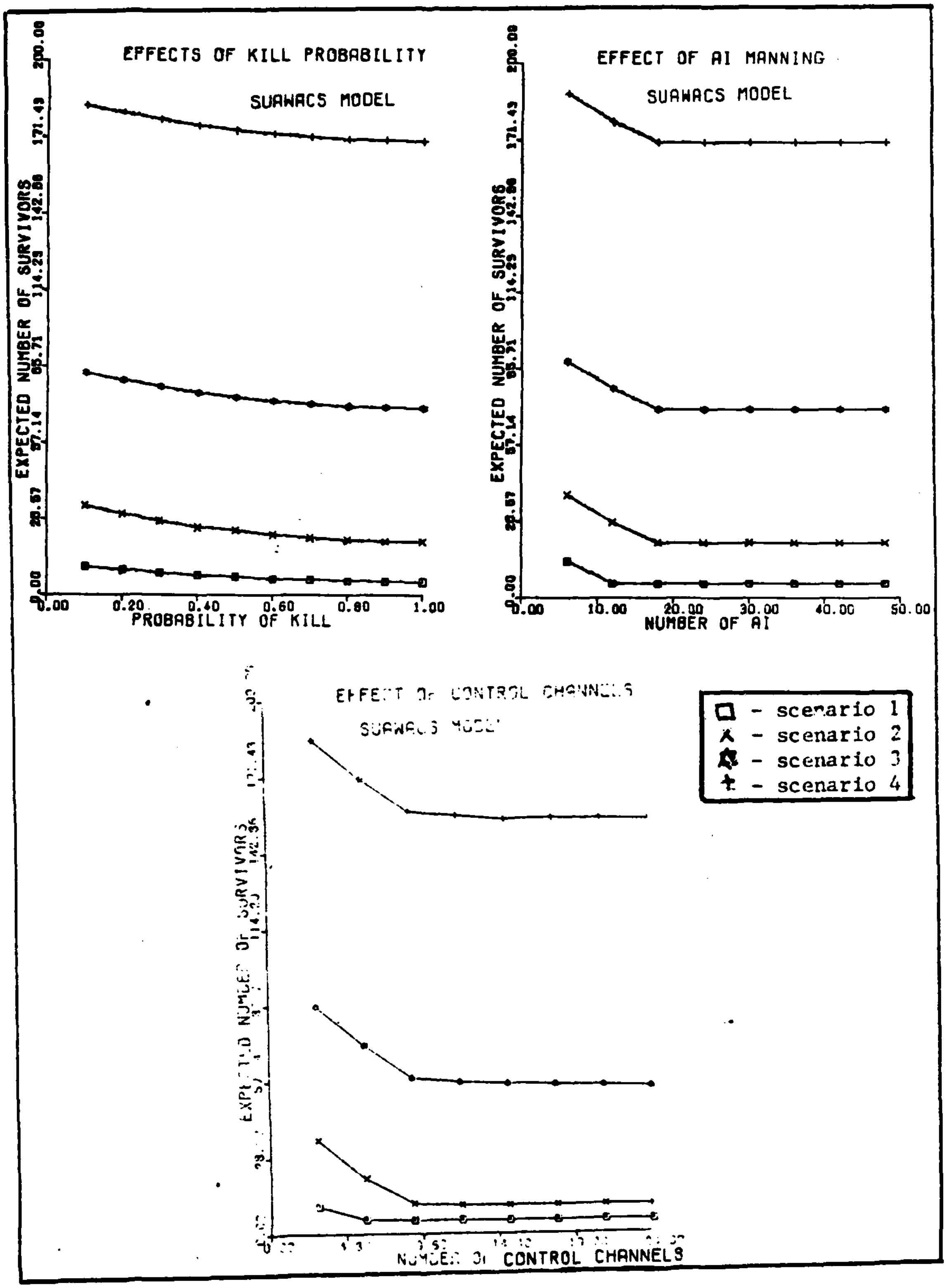

Figure 29. SUAWACS Model Results

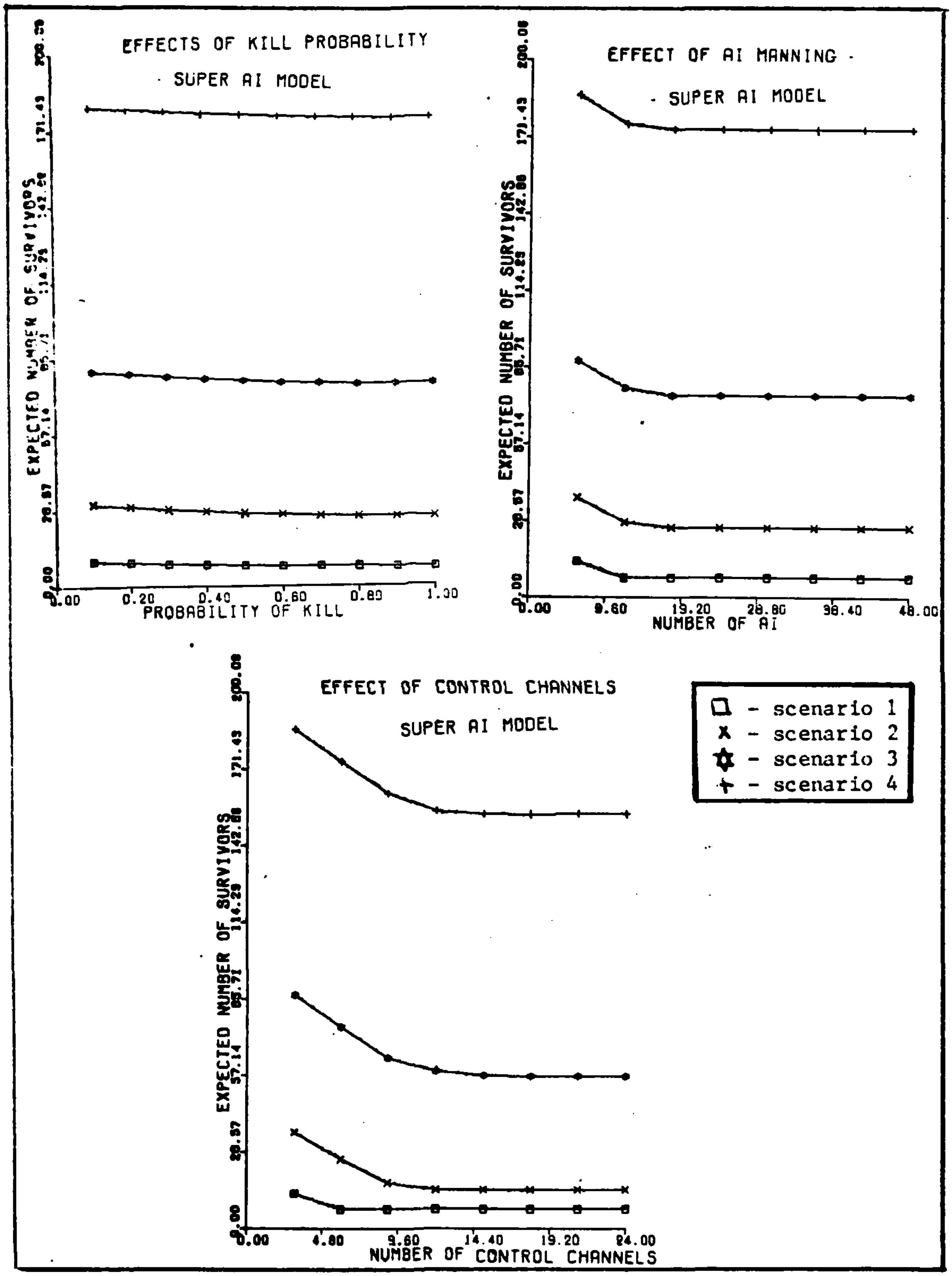

Figure 30. Super AI Model Results

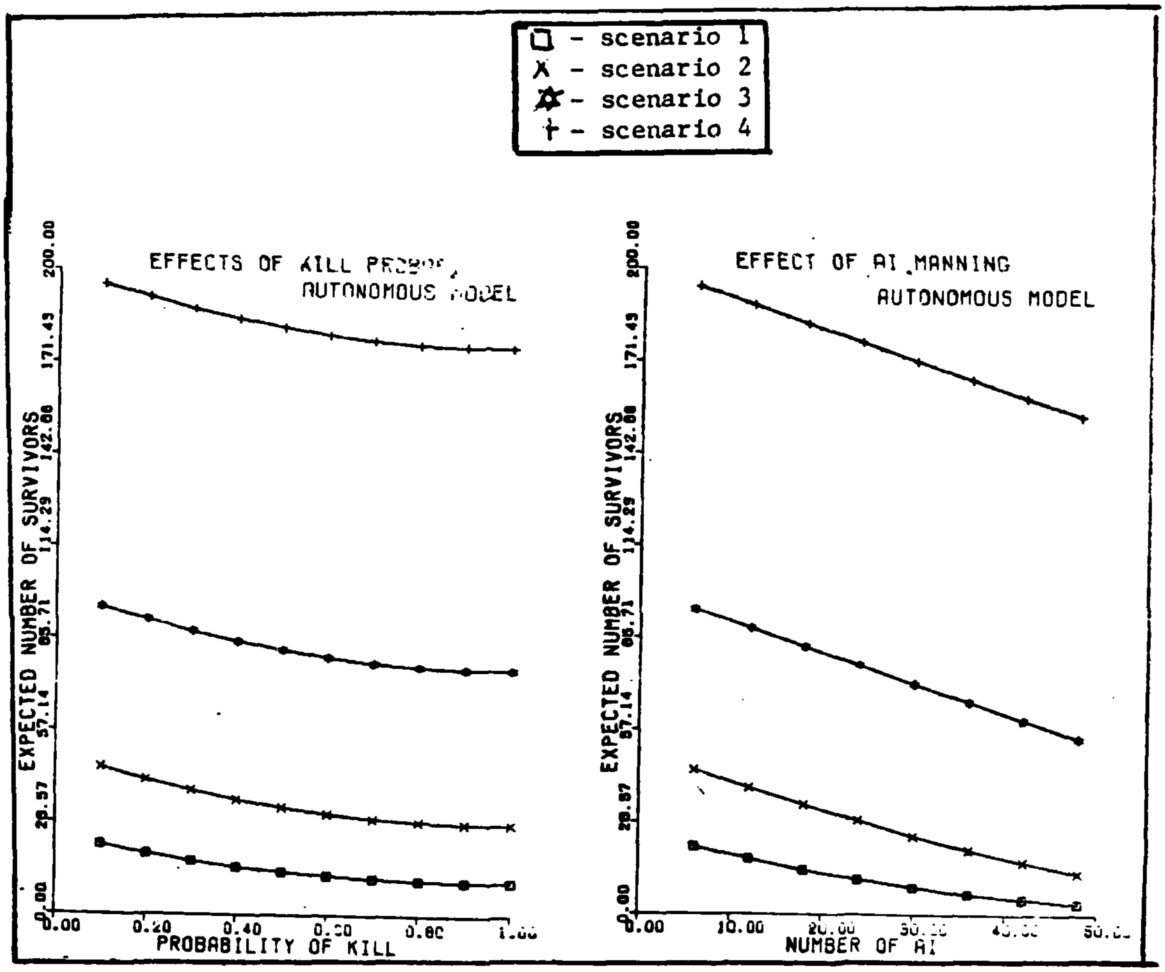

Figure 31. Autonomous MOdel Results

on the parameter behavior.

Figure 32 shows the effect of the size of the penetration threat for the four cases considered as a percent of the total number of penetrators which survive. The advantage of the SUAWACS is the largest in the case of a small penetration force. However, as the force size increases, the difference in the effectiveness between defensive scenarios is very small. Against a small penetrating force, more command and control leads to a more effective defense. However, large penetration force saturates control channels and interceptors. It also alleviates the detection handicap of the autonomous defense by producing a target rich environment, increasing this defense's effectiveness.

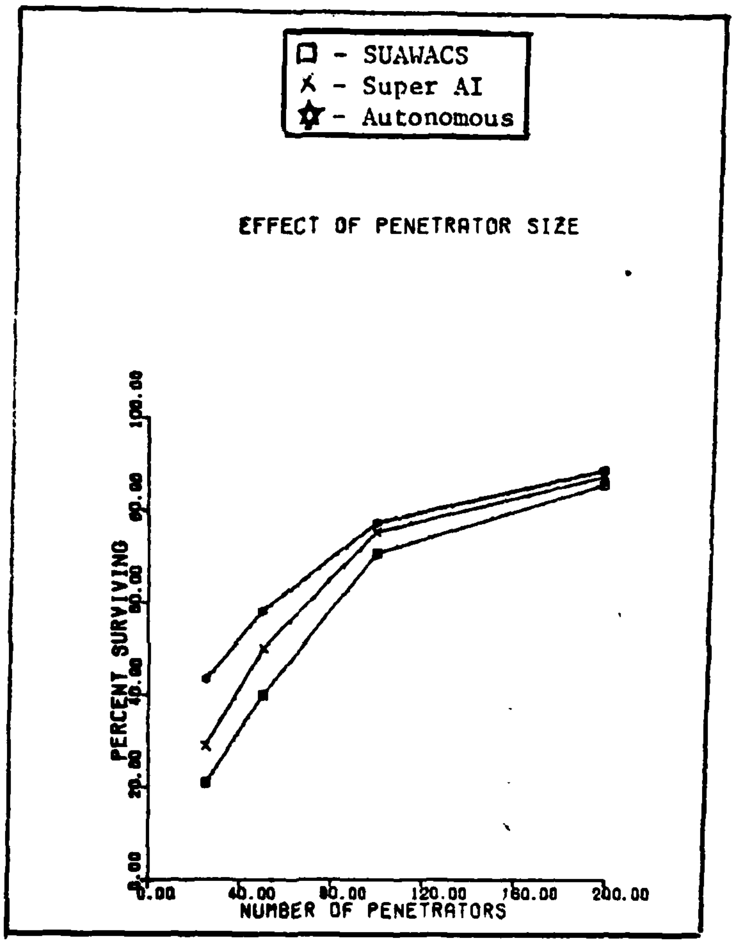

Figure 32. Penetration Size

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

This chapter recaps the models developed in this study and suggests some possible areas for further research. Some of these involve improvements which could be made to upgrade the current model, others are related topics which could expand this set of models.

Summary

The purpose of this study was to examine the effects of command and control on forward air defense. Three defensive postures were proposed as possibly occurring in the FAD. A review of current models, showed they had limited applicability to a single scenario. A new modeling philosophy was developed which provided a fundamental base of assumptions and methods. This was expanded as appropriate to include the effects of the command and control system used in each scenario. A limited example was done using the model to illustrate its potential.

This study provides an important contribution to existing models of the forward air defense. By building on a fundamental set of assumptions and techniques, it allows several C^2 environments to be modeled and comparisons among them made. It also allows explicitly models in parameters which are important to command and control: the number of control channels, and an interceptor availability which changes with time. The model

results are given in terms of the expected number of survivors, rather than the expected number of kills. In the close control cases, this is broken into the cases of penetrators undetected, never assigned and surviving all attacks. The user may use this breakout to easily find the factor which makes the major contribution to the penetrator survival and then look at the parameters which can influence these areas.

Recommendations for Further Study

There are several improvements which can be made to this model. Some involve changes to the current model, other address supplemental models which further examine issues of command and control.

This model allows the interceptors to have two shots. One improvement to the models would be to expand this to the general case of 1,2,3,...,n shots, allowing the user to specify the number of shots. The concept of equivalent AIs was used as a crude method of modeling two shots. An alternative approach implies a nonhomogeneous interceptor pool, with the model keeping track of the expected number of interceptors in the pool with 1,2,3,...,n shots left.

Another area for expansion involves the penetrator force. Currently, a homogeneous penetrator force is assumed. It may be desirable to model a nonhomogeneous penetrator force composed of two segments, possibly representing bombers and cruise missiles. This affects the probability of detection, probability of kill, and times involved in the intercept. A prioritization of

penetrators may take place when interceptor assignments are being made.

The third area for expansion of the current model is a change in the way the close control system handles the interceptor. This model assumes the SUAWACS or super AI stays involved until the intercept is completed. Against a large threat, the control center may release control to the individual AIs at an earlier point, e.g., once the AI acquires the penetrator on its radar. The radar capabilities of the interceptor plays an important role in determining the length of time until the C^2 channel is free.

The assumption that the AIs will not need to leave the FAD to refuel could be relaxed. Interceptor availability would have to be adjusted to reflect the time intervals the AIs are returning to refuel. these interceptors either completely recycle, returning to base to refuel, and then the same AI returns to the FAD; or the AIs which need to refuel are replaced by another AI. The rate at which the interceptors return to refuel is also an important consideration.

Another area for improvement of the model is a better measure of the C^2 cycle time. It is an attempt to quantify and characterize the differences in command and control capabilities. Perhaps a better parameter could be developed which more aptly characterizes these differences.

Further consideration of the autonomous model is also warranted. Much flexibility exists in the division of the grid into rows and columns in this model. The methods for doing this reflect the use of interceptors and assumptions the user is making

concerning their employment and capabilities. Research needs to be done to indicate various methods for division of the grid and the assumptions that are made by each method. This research would examine search theory concepts and employment of interceptors in autonomous situations.

Another autonomous model which could supplement the C^2 scenarios considered would allow communication between the AIs. This could be limited to within cell communication, which would prevent two or more AIs which detect the same penetrator from all going to make the intercept. One AI would attack leaving the other AIs available to attack successive penetrators. The model could also consider the effect of communication from cell to cell. This would allow the interceptors to pass information on penetrators they detect to the AIs in the cell immediately behind them.

A final model which the user might choose to develop is one which generates input values. Search theory methods may be used to find the probability of detection. Also knowledge of the interceptors capabilities and tactics can be used to find the cycle time. Depending on the degree of accuracy and sophistication the user desires, this could be a complex stand alone model or could be incorporated as a series of subroutines to the current model.

Conclusions

This study has resulted in the development of a methodology which can be used to model various C^2 scenarios. Although the

number of scenarios considered was limited, suggestions are provided for expanded scenarios, which, if implemented, could result in a family of models which examine command and control in detail. One of the model's big advantages is the set of underlying principles it presents for developing the expected number of FAD survivors and the expansion of these principles into various C^2 scenarios. These models may be used to determine the best defensive tactics to use against a given threat, or the areas for improvement in forces and future expenditures. They may also be used individually as a sensitivity analysis of the parameters which may impact some other model.

The result of this study is a model which can be used by FTD in exploring these C^2 scenarios. This model provides a basis for modeling C^2 effects, but work still remains. The assumptions made initially are restrictive, and may need to be relaxed for further application. Other refinements of the model can be made to allow a more general application of the model.

The evaluation of different C^2 scenarios and their effectiveness in forward air defense is especially relevant in light of current research on an AWACS "killer." This model allows an evaluation of the value of an AWACS killer in reducing the effectiveness of the defense. It also allows the defense to investigate how to strengthen the defense for each of the command and control scenarios.

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APPENDIX A

A REVIEW OF THE COPEM MODEL

This appendix reviews the COPEM model in detail and examines some of the assumptions made in its development which limit its applicability. This model was a candidate for expansion as a method of modeling C^2 effects. The assumptions which are discussed prevented its application and forced research in this direction to be abandoned. However, a key idea used from this model used in developing the C^2 models presented, is the interceptor availability function.

The COPEM study of forward air defense needed to be concerned with factors which specifically impact command and control aspects, chief among these was interceptor availability. It also sought a fundamental set of concepts which could be used as a basis for building several models with different C^2 philosophies. A review of current models of forward air defense indicated the Stanford Research Institute (SRI) Corridor Penetration Model (COPEM) might serve as an appropriate starting model since it had both features: it dealt with interceptor availability as a function of time and it was a general model with the potential for application to other models. In trying to apply this sophisticated analytic model to the problem, some implicit assumptions which were made in the development of the COPEM model were uncovered which limit the model's applicability to the problem at hand. A review of this model is presented here to clarify it for the interested reader and to emphasize the

assumptions which are made in its development.

The COPEM model is divided into two sections: the forward air defense model and the weapon/target allocation model. Only the air defense portion will be reviewed here. The COPEM model finds the probability a penetrator reaches a certain depth in the forward air defense zone before being destroyed. The forward air defense zone is manned by interceptors from k airbases and controlled by GCI radar. The zone itself is divided into a rectangular grid of cells. The interceptors are distributed across this grid according to some probability distribution, and penetrators enter and fly through the grid in straight lines, parallel to the sides. The number of intercepts which can be made depends on where the penetrator is detected and how many intercept attempts the control center can then make in the time remaining before the penetrator exits with the interceptors available. A major assumption of the models is that this can be represented as a non-homogeneous Poisson process. The parameter $_t$, is time dependent and incorporates all information about the interceptor force which influences the number of interceptors airborne and available for assignment. The parameter is estimated iteratively for discrete time intervals.

<u>Basic Model</u>. The problem of forward air defense (FAD) considers a group of enemy penetrators who are trying to advance through a corridor which is defended by an interceptor force. This model provides a way to generate the probability that an arbitrary penetrator reaches a depth d. The following assumptions

are made to further define the problem:

(1) The bomber force is homogeneous.

(2) Bomber suppressive fire is not included.

(3) All penetrators fly in straight lines through the grid from their point of entry.

(4) Constant velocities are assumed for the penetrator and interceptor.

(5) Attrition of the penetration force is only due to interceptors, no SAMs or AAAs.

(6) The AI force is distributed within the corridor with a specified number airborne at the time the first penetrator arrives.

(7) AIs are vectored to penetrators by the C^2 system.

(8) The probability of detection is a function of depth only.

Consider a penetrator which arrives at time t=0 at the corridor boundary. It survives to a depth d, or equivalently, to a time t, if it has been undetected up to that time, or it has survived all attacks made on it so far. Once the penetrator is detected, engagement attempts will still be made on it until it is destroyed or it flies out of the FAD. The number of engagement attempts which can be made on it depends on the length of time it has to make the attempts.

For each intercept attempt, there is some probability, P_k, that the interceptor will kill the penetrator. To survive t' time

units after detection, the penetrator must survive the i intercept attempts the defense makes. The number of attempts, i, which can be made on a penetrator has some probability associated with it. Let $p_i(t'/t_d)$ be the probability that i intercept attempts can be made on a penetrator within t' time units following detection at time t_d. So the probability of surviving to time t after detection is:

$$P_s(t'|t_d) = \sum_{i=0}^{\infty} (1-P_k)^i p_i(t'|t_d) \qquad (A.1)$$

Surviving an additional t' time units after detection, is the same as surviving to time t where $t=t_d+t'$. So, using the arguments of conditional probability:

$$P_s(t)=\int_0^t P_s(t-t_d|t_d)p_d(t_d)dt_d + \int_t^{\infty} p_d(t_d)dt_d \qquad (A.2)$$

The probability of surviving to time t, is the probability of surviving all attacks made following detection and the probability of not being detected after time t. Substituting equation A.2 into equation A.1 gives:

$$P_s(t)= \sum_{i=0}^{\infty}(1-P_k)^i\int_0^t p_i(t-t_d|t_d)p_d(t_d)dt_d + (1-P_d(t_d)) \qquad (A.3)$$

As mentioned, distance and time are related through the formula, velocity x time = distance. If v_b is the speed of the bomber, than the probability of penetrating to depth d or greater is the same as the proability of surviving to time t. A

substitution of variables in equation 1.3 gives:

$$P_s(t)=P_s(\frac{d}{v_b})=\sum_{i=0}^{\infty}(1-P_k)^i\int_0^{\frac{d}{v_b}} p_i(\frac{d}{v_b}-t_d|t_d)p_d(t_d) + (1-P_d(\frac{d}{v_b})) \quad (A.4)$$

Previously, time of detection, t_d, was measured relative to time zero when the penetrator entered the corridor. In general, the penetrators will arrive at a variety of times, t_a, where $0<t_a<t_{max}$. The detection probability density function depends only on penetration depth or equivalently, the time following arrival. The functions $p_i(t'/t_d)$ depend on when that time occurs since availability of the interceptor force has an impact on their values. So detection time must be adjusted and placed on the relative time scale by adding arrival time to the time of detection for the $p_i(t'/t_d)$ function. The probability of penetrating distance d into the corridor, given the penetrator arrives at time t is:

$$P_p(d|t_a)= \sum_{i=0}^{\infty}(1-P_k)^i\int_0^{\frac{d}{v_b}} p_i(\frac{d}{v_b}-t_d|t_d+t_a)p_d(t_d)+(1-P_d(\frac{d}{v_b})) \quad .5)$$

Multiplying the conditional probability by the probability of arriving at time t_a and integrating over the range of arrival times gives the average penetration probability function:

$$P_p(d) = P_p(d|t_a)p_a(t_a)dt_a \quad (A.6)$$

Substituting as before and using dummy variables of integration, $x=t_d$ and $y=t_a$:

$$P_p(d)=\sum(1-P_k)^i\int_0^{t_m}\int_0^{\frac{d}{v_b}} p_i(\frac{d}{v_b}-x\;x+y)p_d(x)p_a(y)dxdy + (1-P_d(\frac{d}{v_b})) \quad (A.7)$$

Equation 1.7 is the general form for the average penetration probability function which can be applied with appropriate specification of parameters. The density functions for $p_d(\cdot)$ and $p_a(\cdot)$ can be specified for each scenario as appropriate. However, the function $p_i(\cdot|t_d)$ represents a conglomeration of information on the defense's manning at any point in time. It depends on a variety of factors, such as availability and location of the fighters at time t.

A fundamental assumption of this model is that the $p_i(\cdot|t_d)$ is a time dependent Poisson process with parameters which are different for each time interval. The COPEM model calls the functions $p_i(\cdot|t_d)$ intercept functions. The authors assume the underlying stochastic process which generates the intercept function is:

$$p_i(t'|t) = \frac{(\lambda_t t')^i\ e^{-\lambda_t t'}}{i!} \qquad (A.8)$$

where $_t$ is dependent on time and parameterizes the intercept intensity function. This can be substituted into the average penetration probability function to obtain:

$$P_p(d) = \sum^{\infty}(1-P_k)^i\int_0^{t_m}\int_0^{\frac{d}{v_b}} \frac{[\lambda_{x+y}(\frac{d}{v_b}-x)]^i\ e^{-\lambda_{x+y}(\frac{d}{v_b}-x)}}{i!} p_d(x)p_a(y)dx\,dy$$

$$+ (1-P_d(\frac{d}{v_b}))$$

Notice that the integrand, can be rewritten by recognizing it as a power series expansion of e^{i}.

$$e^{-\pi\lambda_{x+y}(\frac{d}{v_b} - x)} = \frac{[(1-P_k)\lambda_{x+y}(\frac{d}{v_b} - x)]^i \; e^{-\lambda_{x+y}(\frac{d}{v_b} - x)}}{i!} \qquad (A.10)$$

Equation A.9 then can be written as:

$$P_p(d) = \int_0^{t_{max}} \int_0^{\frac{d}{v_b}} e^{-\pi\lambda_{x+y}(\frac{d}{v_b} - x)} p_d(x) p_a(y)\,dx\,dy + [1-P_d(\frac{d}{v_b})] \qquad (A.11)$$

The problem now requires the value of λ_t which is the parameter of a nonhomogeneous Poisson process and is a function of time.

<u>Estimating the Intercept Function</u> Since λ_t is the parameter of a nonhomogeneous Poisson process, the assumptions of the Poisson process will be reviewed for the appropriate application to this problem. A Poisson process is basically a counting process, which counts the number of events which occur in an interval (0,t'). Four assumptions must be met for a Poisson process to exist.

(1) $N(0) = 0$

(2) Independent increments

(3) $P[N(t+h) - N(t) = 1] = \lambda(t)h + o(h)$

(4) $P[N(t+h) - N(t) \geq 2] = o(h)$

where a function is o(h) if $\lim_{h \to 0} \frac{f(h)}{h} = 0$. This implies that for h small, the function f(h) is small compared to h. To use this model for forward air defense, these assumptions must fit our

problem. The first assumption requires that no intercepts will have been made at time of detection. This is reasonable for the assumption of a control radar net which makes the detections of penetrators.

The second assumption requires that the process be independent of everything which has previously occurred. Over a discrete time increment, the number of intercepts will basically be a function of how many interceptors there are and where they are located relative to the penetrators. Although this depends on what has happened before, for any given time interval t, the parameter λ_t takes this information into account, making the process itself independent of time. Multiplying λ_t the average rate at which interceptions occur in interval t, by a length of time t' less than t, gives the average number of intercepts which could occur in time t'. In the FAD, for a fixed level of resources and penetrators, the probability of interception will be the same for a fixed length of time.

The third assumption requires that the probability that one interception occurs in some small time interval, h, is equal to the mean intercept rate times the length of time available to do it plus o(h). This is reasonable for this problem, there is some average time to intercept, or inversely some average number of intercepts which can be accomplished over a given period of time.

The fourth assumption requires some time to elapse between interceptions, no two can occur simultaneously on the same bomber. In a controlled environment, this would require the control system to make two separate and distinct assignments against a

penetrator. With the advantage provided by control and knowledge, a second assignment would not be made until the outcome of the first was known. It would be a waste of resources.

These are the four assumptions underlying this problem. They are reasonable for the forward air defense scenario as defined. It is important to remember the time dependence of the parameter is what allows this process to be used. The parameter itself takes into account the factors which changes value over time, and allows the assumptions of the Poisson process to hold. The problem now is to estimate λ_t, in order to parameterize the intercept functions.

The intercept intensity function represents the mean rate of intercepts which can be made against this bomber over time t. For some interval, t', $\lambda_t t'$ is the expected number of intercepts possible against the penetrator. It depends on availability and location of the fighters and their system performance capabilities. Under the assumption that intercepts, given detection, occur in accordance with a Poisson process, then the time to first intercept is a random variable distributed exponentially with parameter λ_t. Recall that a Poisson process has exponentially distributed interarrival times and that λ is the parameter of both the Poisson and exponential distributions. The mean of the exponential distribution is the reciprocal of its parameter, so if an estimate for the mean time to first intercept can be obtained, it can be inverted to give an estimate of the parameter λ_t.

<u>Frame of Reference</u>. The expected time to first intercept

is equal to the time to intercept a penetrator multiplied by the probability of making the intercept. The penetrators are moving through the corridor with various arrival times and entry points, while the AIs themselves may be located in several places. The time to intercept will depend on what pairings of interceptors and penetrators are made. The frame of reference and method used to make these calculations is based on the locations of penetrator and AI on a grid and is described below.

The forward air defense zone or corridor which the penetrators will use is divided into a grid over which the fighters are to be used. (see Figure 33) The grid has r rows and c columns. Penetrators will enter at random along the zone and fly in straight lines until they are either destroyed or exit. A penetrator when detected, will be considered to be in the center of its cell. The interceptors are also considered to be in the center of their cells, even though they are orbiting within the cell in a search pattern.

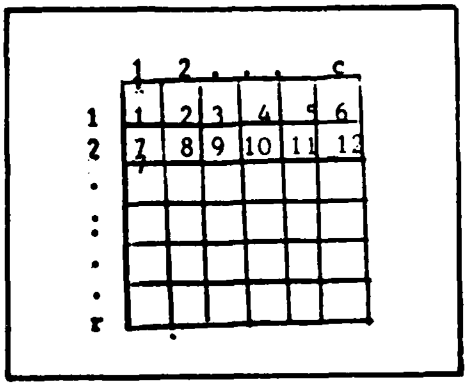

Figure 33. Grid Structure

The time to intercept a penetrator detected at some cell i, is dependent on the availability and location of the interceptors. This in turn is influenced by a variety of factors, including time to recycle/replace AIs, the time and manner of the penetration attack, etc. The defense will try to intercept a penetrator as quickly as possible. They will use interceptor/penetrator pairings which will have the smallest intercept time, T_{ij}.

The time to intercept is dependent on the time required for data processing, T_A, the time to fly there, T_F, the time for the AI to search and acquire the penetrator on its radar, T_S, and the time to convert and fire, T_L. Three of these, T_A, T_S and T_L are essentially constant. T_F is the only one dependent on the positions of the penetrator and the fighters on the grid. Since both the penetrator and the AI are assumed to be located in the center of their respective cells, then T_F can be calculated using the geometry of the situation with elementary algebra and trigonometry. The mathematics will not be shown here.

The defense wishes to send the interceptor with the shortest T_{ij}. There will not always be an interceptor in every cell, the second or third or an even later choice might be used depending on AI availability at the time of detection. If no one is available, the penetrator can advance to the next cell before he is reconsidered by the control system for reassignment.

Hence, for a penetrator detected in cell i at time t, each cell in the grid must be checked to determine if there is an interceptor there at time t and that the interceptor is available for assignment to the penetrator which occurs with some

probability. Assume the probability, p_i, that a fighter is placed in some cell i initially is known for all $i \in I$. A fighter may be unavailable for assignment if he is already assigned to a penetrator. How often he is used depends on how short his interception time to the penetrator in cell j is when compared with the other fighters, how often a penetrator is detected in cell j, and how many penetrators are in coverage which require an interceptor assignment. The first consideration, is how his T_{ij} compares with the other interceptors. A convenient method for dealing with this involves ranking the time to intercept the penetrator detected in cell j from each of the i cells. Let R_{ij} be the rank of cell i relative to cell j in terms of the time it would take a fighter from cell i to intercept a bomber first detected in cell j. R_{ij} will equal 1 for the i which has the minimum T_{ij}, 2 for the i with the next smallest T_{ij} and so on. With the preliminary definitions out of the way, an estimate for the expected time to first intercept can be made. Define:

n_k - the total number of fighter interceptors at airbase k

$n_k(t)$ - expected number of interceptors from base k that are airborne and unassigned at time t

p_{ik} - probability that an arbitrary airborne interceptor from base k is in cell i;

$p_{ik}(t)$- probability that there is an interceptor from base k in cell i at time t;

$q_j(t)$ - probability that there are no interceptors available to be assigned to a bomber in cell j at time t:

$q_{ij}(t)$- probability that an interceptor assigned to a bomber in cell j at time t is from cell i:

$Q_{ij}(t)$- probability that there is an interceptor in cell i that can be assigned to a bomber in cell j at time t;

R_{ij} - ranking of cell i relative to cell j in terms of the time it takes a fighter from cell i to intercept a bomber first detected in cell j

T_c - bomber flight time between calls;

T_{ij} - expected time to the first intercept given that an interceptor from cell i is assigned to a bomber in cell j;

δ_{ijk} - indicator variable equal to 1 if an interceptor from airbase k is in cell i is capable of intercepting a bomber in cell j and 0 otherwide

For $N_k(t)$ interceptors from base k airborne and unassigned at time t, the probability there is an unassigned AI available from base k at time t is:

$$P^*_{ik}(t) = 1 - (1-p_{ik})^{N_k(t)} \tag{A.12}$$

Only the expected value of $N_k(t)$ is known denoted by $n_k(t)$ so only the average probability can be found:

$$P_{ik}(t) = E[P^*_{ik}(t)] = E[1-(1-p_{ik})^{N_k(t)}] \tag{A.13}$$

To simplify this expression the author assumes $N_k(t)$ is binomially distributed with parameters $p = n_k(t)/n_k$ and $N=n_k$, so the expression can be written:

$$P_{ik}(t) = 1 - (1-(p_{ik}n_k(t)/n_k))^{n_k} \qquad (A.14)$$

Then the probability there is an interceptor in cell i that can be assigned to a bomber detected in cell j at time t is:

$$O_{ij}(t) = 1 - \prod_{k=0}^{K} (1-\delta_{ijk}P_{ik}(t)) \qquad (A.15)$$

where K is the total number ofinterceptor bases.

Let i_m be the value of i such that the rank of cell i is m. Since the grid has r rows and c columns, and a total set of I cells, m=1,2...,I. For example: 10_1 means the 10th cell has the smallest interception time T_{10j} to cell j and is ranked number one of all the fighters for assignment. Now the probability that the interceptor assigned to a bomber in cell j is from cell i can be calculated as:

$$q_{i_m j}(t) = [1-\sum_{x=1}^{m-1} q_{i_x j}(t)]Q_{i_m j}(t) \quad \text{and } q_{i_1 j}(t)=Q_{i_1 j}(t) \qquad (A.16)$$

The probability that no AIs are available to assign to the bomber in cell j at time t is:

$$q_j(t) = 1 - \sum_{x=1}^{I} q_{i_x j}(t) \qquad (A.17)$$

The expected time to first intercept of a bomber detected in cell j at time t is broken into two parts: the expected time to intercept given an AI is available at time t, times the

probability an AI is available, and the expected time to intercept if an AI is unavailable at time t, times the probability an AI is unavailable.

$$E[T_j(t)] = E_1[T_j(t)] + E_2[T_j(t)]$$

$$= E_1[T_j(t) | AI]P(AI) + E_2[T_j(t) | \overline{AI}]P(\overline{AI}) \quad (A.18)$$

where $E_1[T_j(t)] = \sum_{i=1} q_{ij}T_{ij}$

The value of $E_2[T_j(t)]$ still needs to be derived. If an AI is unavailable the penetrator flies forward using time T_c and has a new expected time to first intercept at this new time t and to this new position. If no AIs are available at this new time t, the penetrator again advances. The argument follows recursively through each time period.

So the model uses this expression:

$$\begin{aligned} E_2[T_j(t)] &= q_j(t)(1-q_j(t))(T_c+E_1[T_j(t)]) \\ &+ q_j(t)^2[1-q_j(t)][2Tc + E_1[T_j(t)]] \\ &+ \dots \\ &+ q_j(t)^m[1-q\ (t)][mT_c+E_1[T_j(t)]] \\ &+ \dots \end{aligned} \quad (A.19)$$

This expression fails to account for the new position at the next time as the penetrator advances and it also assumes the sum is finite which does not account for penetrators which penetrate safely beyond the edge of the forward air defense zone. Adjustments and corrections are noted by Clemens in Ref. 5. For consistency and continuity, these changes will not be discussed in

detail here, and the original derivation will be used.

The expression for E_2 can be rewritten using two identities:

$$\sum_{i=1}^{\infty} x^i = \frac{x}{1-x} \quad \text{and} \quad \sum_{i=1}^{\infty} i x^i = \frac{x}{(1-x)^2} \tag{A.20}$$

as:

$$E_2[T_j(t)] = q_j(t)E_1[T_j(t)] + T_c q_j(t)/(1-q_j(t)) \tag{A.21}$$

which can be added to $E_1[T_j(t)]$ to get an expression for the expected time to intercept for a penetrator in location j at time t.

$$E[T_j(t)]=(1+q_j(t))E[T_j(t)] + T_c q_j(t)/(1-q_j(t)) \tag{A.22}$$

Finally, summing over all j cells where detection of a penetrator may occur, gives the expected time to intercept at time t.

$$E_t[T] = \sum_{j=1}^{I} P_j(t)E[T_j(t)] \tag{A.23}$$

where $P_j(t)$ is the probability a bomber is detected in cell j at time t. The reciprocal of $E_t[T]$ is the desired estimate of λ_t.

Estimating $n_k(t)$ In finding the solution to this problem, one final parameter needs to be defined: the expected number of fighters from airbase k available for assignment as a function of time. The basic idea is to iteratively modify availability to reflect the events of preceeding intervals. If all AIs are always available, then the graph would be a constant. First the graph is modified to account for the AIs refueling, then this new curve is modified to reflect interceptor assignments and the time required to make the intercept for penetrators which enter. Penetrator entry is considered in successive intervals and the curve is adjusted after each interval, allowing a new estimate of λ_t to be made based on the newly modified curve. To illustrate define:

Δt - time interval between successive estimates of λ_t;

T_i - ith time interval for the calculation of λ_t; T_i is the interval $((i-1)\Delta t, i\Delta t)$

n_k - total number of fighters stationed at base k;

$N_d(T_i)$ - the number of bombers detected in the time interval T_i;

$N_k(T_i)$ - the number of fighters from base k assigned to bombers detected in the time interval T_i;

$P_k(T_i)$ - the probability that a bomber detected in the time interval T_i will have a fighter from base k assigned to it;

T_f - total time a fighter can remain airborne before recycling;

T_r - the minimum time for a fighter to recycle

r_s - the scramble rate at which fighters take off, measured in fighter/unit time.

Before penetrators begin entering the FAD, assume the number of interceptors is maintained at some constant level, $n_k(0)$. The maximum value, $n^*(0)$ is:

$$n_k^*(0) = \frac{T_f}{T_f + T_r} n_k \qquad \text{(A.24)}$$

At time t=0, the remaining AIs on the ground all are scrambled. The number of AIs increases linearly with the rate of increase equal to the scramble rate, r_s. All the AIs will be airborne at time t_1:

$$t_1 = \frac{n_k - n_k(0)}{r_s} \qquad \text{(A.25)}$$

At time t_2, the AIs begin to recycle at the same rate that they scrambled for a length of time T_r, the time required by one AI to recycle. If there are AIs still leaving to recycle, then the number of AIs will stabilize since there is a one for one exchange of AIs leaving to recycle and those returning from recycling. If the AIs have all left to recycle when the first AI returns, then the number of AIs increases linearly in accordance with the scramble rate. This recycling process repeats as necessary for the length of time the penetration lasts. Figure 34 illustrates this process for $n_k(0)$=20, n_k=30, r_s=1/min, T_f=50 min and T_r=20.

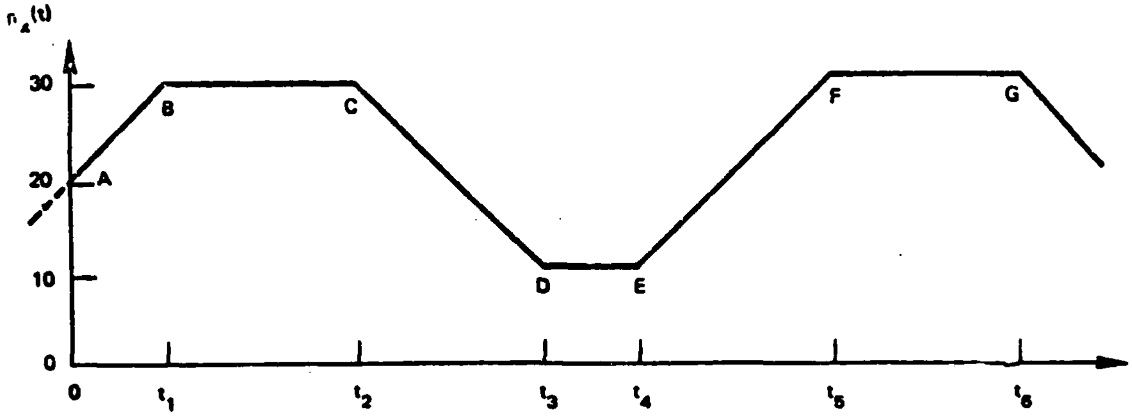

Figure 34. Modification of $N_k(t)$ For First Interval

Now consider what happens as penetrators enter and interceptor assignments are made. The method for modification will be illustrated using the example presented in Figure 34. Estimates of the intercept intensity function λ_t are computed for 10 minute intervals. An estimate of λ_0 as specified by the user is 14 minutes. The expected number of bombers detected in this interval is found by convoluting the detection time and arrival time distributions. Say $N_d(T_1)=7$ bombers are detected, and there is a 5/7 probability that the penetrators will have a fighter from airbase k assigned to them. The expected number of fighters from airbase k assigned to attack bombers in this interval is:

$$N_k(T_1) = P_k(T_1)N_d(T_1) = (5/7)7 = 5 \tag{A.26}$$

So in the first ten minute interval, five fighters from base k are assigned to penetrators. The number of fighters from base k available for intercept at time t = 10 minutes has been decreased

by five. This is point A in Figure 35. The probability of kill is .7. So the expected time to successful intercept (a kill) is 14/.7 or 20 minutes.

Thus at time t=20, all AIs will rejoin their unassigned fighter forces. This is indicated by the line segment connecting points B and C in Figure 35. This completes the modifications of the curve due to events which occurred in the first 10 minute interval. At this point, λ_{T_2} can be calculated.

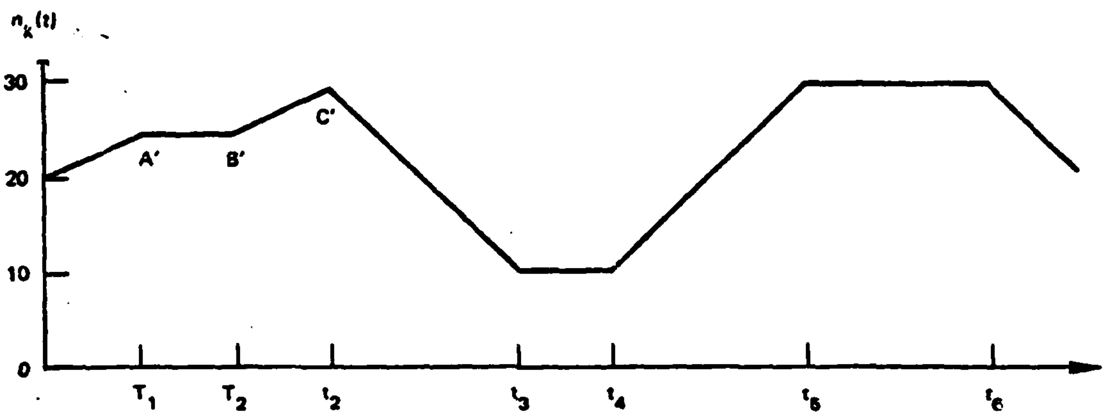

Figure 35. Modification of $n_k(t)$ for Second Interval

This procedure is repeated for times $T_2, T_3, \ldots, T_n$. After each time, λT_{i-1} the previous estimate of the intercept intensity function is used to find the expected time to a successful intercept. The availability curve can then be modified for the activity in that time interval and a value for λ_t is calculated.

After all computations of λ_t are made for each interval T_i, there is a set of λ_T. Numerical interpolation can be applied to find the values of λ_T between the estimates which were derived.

Limitations of the Assumptions. A few of the basic assumptions of the COPEM model which are not clearly stated are found in this estimation procedure for $n_k(t)$. These assumptions imply a great deal about the behavior and use of the interceptors which was not made clear early in the model development.

In calculating the expected time to successful intercept, or a kill, the expected time to first intercept is divided by the probability of kill. This situation represents a geometric distribution for kill: there is success of failure on each trial (kill/no kill), independent trials, and repeated trials until a success, a kill, occurs. The parameter of the geometric distribution is λ, in this case .7, and the mean is $1/\lambda$ or 1/.7. The expected time to a successful intercept is the mean time to make an intercept at time t, 14 minutes, times the mean number of trial required until a kill occurs, 1/.7, which equals 13/.7 or 20 minutes.

This calculation implies that once an interceptor is assigned to a penetrator it makes repeated attempts against the penetrator until a kill is achieved. This implies the AI has enough weapons to make the required number of attacks, using one weapon each attack. It also implies that each separate attack made by the interceptor requires the same average time which was originally calculated including the time to fly there and data processing as well as time to search and acquire. Certainly, in repeated attempts, the AI is not required to reaccomplish steps requiring times T_A and T_S. In addition, the interceptor's time to fly to the penetrator could have been sizeable and now is much smaller

since the AI is in the immediate vicinity for each attack.

One other subtle point, which has important implications, is the fact that the AIs are allowed to rejoin the interceptor force after completing an assignment. Thus the AIs never recycle to rearm, until they return for fuel. If a weapon has a low P_I against these penetrators, then the AI will use several weapons to achieve a successful intercept. If there are many penetrators and a small interceptor force, the AI will probably be reassigned as soon as he completes an intercept attempt. If the expected time to intercept is short or the AI can remain airborne a long time; then he makes several intercept attempts before returning to refuel. Consider an example with P_k=.3 and the AI having 7 intercepts assignments before returning to refuel. The AI will use 2.5 weapons on each intercept, and will use a total of 7(2.5) or 17.5 weapons before returning to rearm! This is a requirement no interceptor can meet. Also since intercepts occur at high speeds, fuel will be used up and refueling may be needed at an earlier time.

The assumptions of the model are not bad, but are not appropriate for all scenarios. These are two assumptions which have become increasingly important with the changes in the composition of the penetration.

Appendix B

Development of Flying Times

This appendix provides the mathematical derivation of the time for an intercepter to fly to a penetrator. The SUAWACS and Super AI models developed require this time since it is part of the time required to make the intercept. In those models, the AIs fly from a CAP position, whose reference point is the center point of the grid. The general development provided below allows the interceptor to be initially located in any cell, not just the CAP position.

Consider a penetrator which is initially detected at cell (x_j, y_j) and has assigned to it an interceptor in cell (x_i, y_i) for for an engagement attempt at some time, t_i. It will take some time, t_f, for the fighter to fly to the intercept point. Hence, at $t_e = t_i + t_f$, an engagement will take place. During time t_f the penetrator has been flying forward at a speed v_b and the fighter has been flying at a speed v_{ai}. Let θ be the fighter's heading relative to the penetrator's flight path, then using the geometry, the time required for the fighter to reach the penetrator t_f can be determined.

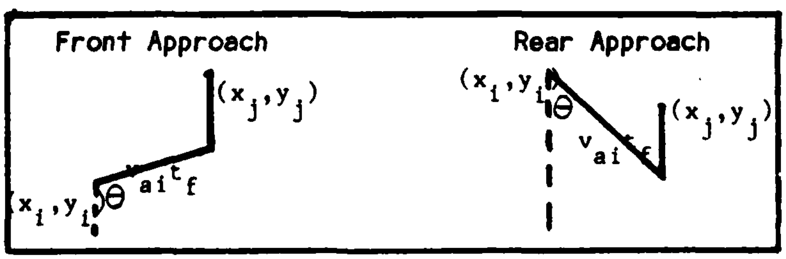

Figure 3 . Intercept Approaches

Let $x_b(t)$ be the position of the penetrator at time t and let $x_{ai}(t)$ be the position of the airborne interceptor at time t. The penetrator is detected at t_d and the position of the penetrator is: $x_b(t)=x_j$ $y_b(t)=y_j$ while the position of the interceptor is: $x_{ai}(t) = x_i$ $y_{ai}(t)=y_i$. For an engagement attempt to occur the penetrator and the interceptor must be at the same point at time t_e: $x_{ai}(t) = x_b(t) = x_j$ (B.1)

$$y_{ai}(t) = y_b(t) = y_j + v_b t \quad \text{(B.2)}$$

Using the geometry:

$$x_j - x_i = x_i - v_{ai} t_f \sin \quad \text{(B.3)}$$

$$v_j + v_b t_f = y_i + v_{ai} t_f \cos \quad \text{(B.4)}$$

Equation B.3 can be rewritten as: $\sin\theta = \dfrac{x_j - x_i}{v_{ai} t_f}$ (B.5)

Using the identity, $\sin^2 + \cos^2 = 1$, obtain:

$$\cos^2 = 1 - \sin^2 = 1 - \frac{(x_j - x_i)^2}{v_{ai}^2 t_f^2} = \frac{v_{ai}^2 t_f^2 - (x_j - x_i)^2}{v_{ai}^2 t_f^2} \quad \text{(B.6)}$$

Substituting this into equation B.4

$$(y_j - y_i)^2 + t_f^2(v_b^2 - v_{ai}^2) - 2v_b t_f (y_j - y_i) + (x_j - x_i)^2 = 0 \quad \text{(B.7)}$$

Note that this equation is a quadratic function of t_f only, so its solution is:

$$t_f = \frac{v_b(y_j - y_i) \sqrt{v_b^2(y_j-y_i)^2 - (v_b^2 - v_{ai}^2)((y_j-y_i)^2+(x_j-x_i)^2)}}{v_b^2 - v_{ai}^2} \quad \text{(B.8)}$$

Appendix C

Derivation of the Autonomous Model

The appendix provides an explanation of and derivation for the autonomous model. For brevity and continuity of Chapter III it excluded in the model development. It is provided here in detail for the interested reader.

A cell may contain one, two, or more interceptors. If there is more than one, but these interceptors do not communicate with each other, and they act independently of each other. The probability of detection of a single penetrator by an interceptor is known. If NP penetrators pass through this cell each time interval, then the probability a single interceptor detects at least one penetrator is:

$$1 - (1-P_d)^{NP} \qquad (C.1)$$

where P_d is the probability of detection.

Note that the number of detections that can be made is limited by the number of interceptors rather than by the number of penetrators. If there are three interceptors and one penetrator, there may be 0, 1, 2, or 3 detections made. The number of detections which can be made represents the number of interceptors which make detections. Once an interceptor makes a detection he immediately takes action towards an intercept. If two interceptors detect the same penetrator, both will begin the intercept attempt, since there is no communication. However, at

the time of actual attack, they will be able to visually identify each other and mutually agree as to which one will make the attempt.

This assumes the two AIs make the detection at approximately the same time, and will arrive to make the intercept against the penetrators at the same time. Only one attempt is made on a penetrators at each time period. In this model, once an interceptor is waved off the penetrator, it resumes searching and does not make a follow-on attempt.

The number of kills is limited by the physical realities of the forces. The largest number of kills which the interceptors can accomplish at any time is equal to the number of interceptors, assuming one kill per interceptor. The number of kills is also limited by the number of penetrators, since there can't be more kills made than there are penetrators to kill. For example, if there are five penetrators and two interceptors, the maximum number of kills which can occur is two. If there are three penetrators and five interceptors, the maximum number of kills possible is three. Hence, the number of kills is limited by the smaller of the number of penetrators and interceptors, min(NP,NAI). If there are more penetrators than interceptors, then the number of kills which can be made is equal to the number of detections the interceptors make. If there are more interceptors than penetrators, there may be duplicate detections of penetrators by the interceptors, but the maximum number of kills which can be made is equal to the number of penetrators.

In the generalization of the fundamental concepts which underlie all the models the key is to find the expected number of penetrators at each interval. To apply this concept to this scenario consider a single cell with NP penetrators at some time interval. By computing the expected number of detections and the kills which are then made against the penetrators, an estimate of the number of surviving penetrators can be made. These surviving penetrators will enter the next cell at the following time interval. Hence, each cell must be considered separately. To find the number surviving, the number of detections and then kills must be computed.

The underlying philosophy of the autonomous scenario C^2 policy makes this computation somewhat difficult from the previous models. In those, once a penetrator was detected, he was considered for assignment at each successive interval after detection. In the autonomous situation this is no longer true, since the information on the penetrator's location is not retained by the system or forwarded to the next interceptor but must be reacquired by the interceptors at each time interval. In this model there is only one class of penetrators considered at each interval, those surviving, since all knowledge of detection is lost.

The development of the distribution of surviving penetrators involves three aspects: the number of interceptors which make detections, the resulting number of attacks, and the number of kills. Consider the case of two penetrators and two interceptors.

There may be zero, one, or two kills which occur. Since there are two penetrators, there may be a maximum of two killed and the interceptor force is capable of making two intercepts. For I kills to occur there must be I or more detections, but with only I kills occurring. The maximum number of detections which occur is limited by the number of interceptors. For I kills, where I is equal to 0, 1, 2 for this example, the distribution develops as folows:

$$P(0 \text{ kills}) = P(0 \text{ detects}) + P(1 \text{ det} \cap 0 \text{ kills}) + P(2 \text{ det} \cap 0 \text{ kills}) \qquad (C.2)$$

$$P(1 \text{ kills}) = P(1 \text{ det} \cap 1 \text{ kill}) + P(2 \text{ det} \cap 1 \text{ kill}) \qquad (C.3)$$

$$P(2 \text{ kills}) = p(2 \text{ det} \cap 2 \text{ kill}) \qquad (C.4)$$

For two kills, there must be two detections which both result in kill; for one kill, there must be one or more AIs which make detections which result in only one kill. If there is one detection it must result in a kill, but if there are two detections, only one can result in a kill.

Now, consider more closely the case of two detections. Since the two interceptors are independent, they can detect the same penetrator or different penetrators. For instance, denote the penetrators by A and B, and the interceptors by 1 and 2. Then the case of two detections can be represented as follows:

CASE 1: Interceptors 1 and 2 see the same penetrator, result 1 attack

a. 1 and 2 see A

b. 1 and 2 see B

CASE 2: Interceptors 1 and 2 see different penetrators, result 2 attacks

a. 1 sees A

   2 sees B

b. 1 sees B

   2 sees A

The probability of both interceptors of only one attack is:

$$\binom{2}{1}\,(1/2)^2(1/2)^0 = 1/2 \tag{C.5}$$

The factor (2 choose 1) counts the two ways to see just one penetrator, either A or B. Either one is equally likely to be seen, or chosen by an interceptor, with probability 1/2, but both interceptors must see the same penetrator, $(1/2)^2$, and not see the other one $(1/2)^0$. The probability the interceptors detect different penetrators is:

$$\binom{2}{1}(1/2)^1(1/2)^1 = 1/2 \tag{C.6}$$

The factor (2 choose 1) counts the two ways, case 2a and case 2b, the interceptors can make two attacks. One of the interceptors will detect one of two penetrators with probability 1/2, but once it has chosen, the other interceptor must see the other penetrator. The probability of detecting any penetrator is equally likely.

This problem becomes interesting when the number of penetrators and the number of interceptors differ. This is actually the classical occupancy problem, where the number of

interceptors and penetrators represents balls and cells respectively. Since more than one interceptor may be assigned to a penetrator, some penetrators may not be attacked. This corresponds to placing r balls in m cells; since more than one ball may be placed in a cell there is a probability that some cells may be empty. The cell occupancy problem is concerned with finding the probability of k empty cells. In this application, the k empty cells corresponds to penetrators which are not assigned interceptors. The solution for the form of this distribution provides the number of penetrators which are unassigned, and can be used to find the number of attacks made.

Once the number of attacks is determined, the number of kills which result can be calculated. If the two detections are of the same penetrator, then only one attack will be made, since one of the two will be waved off. Only if the two detections are of two different penetrators can two attacks be made. Hence, if one kill is being considered, two detections result in only one kill when (a) the two detections are of the same penetrator or (b) the two detections are of different penetrators but only one kill occurs.

$$\binom{1}{1}P_k \binom{2}{1}(1/2)^2(1/2)^0 + \binom{2}{1}P_k(1-P_k) \binom{2}{1}(1/2)^1(1/2)^1 \quad (\text{:}.7)$$

If there is only one attack made and one kill is required then the attack must result in a kill. If there are two attacks made and just one kill required, then only one attack results in a kill. But with two attacks made on penetrators A and B, then either A is killed and B survives or B is killed and A survives. So there are

two ways for this to occur.

Now all aspects of the problem have been addressed; the result for this example is:

$$
\begin{aligned}
P(0 \text{ kills}) = {} & (1-P_d)^4 + \binom{2}{1}(1-(1-P_d)^2(1-P_d)^2\binom{1}{0}(1-P_k)^1 \\
& +\binom{2}{2}(1-(1-P_d)^2)^2 \quad \binom{1}{0}(1-P_k)\binom{2}{1}(1/2)^2(1/2)^0 \\
& + \binom{2}{0}(1-P_k)^2\binom{2}{1}(1/2)(1/2)
\end{aligned}
\tag{C.8}
$$

$$
\begin{aligned}
P(1 \text{ kills}) = {} & \binom{2}{1}(1-(1-P_d)^2)(1-P_d)^2 \quad \binom{1}{1}P_k(1) \\
& + \binom{2}{2}(1-(1-P_d)^2) \quad \binom{1}{1}P_k\binom{2}{1}(1/2)^2(1/2)^0 \\
& + \binom{2}{1}P_k(1-P_k)\binom{2}{1}(1/2)(1/2)
\end{aligned}
\tag{C.9}
$$

$$
P(2 \text{ kills}) = \binom{2}{2}(1-(1-P_d)^2)^2 \quad \binom{2}{2}P_k^2 \ \binom{2}{1}(1/2)^2(1/2)^0 \tag{C.10}
$$

In summary, this problem is considered in three phases: detections, attacks and kills. The number of detections which can be made is binomially distributed with parameters n and p where n is the number of AIs, and p is $1-(1-P_d)^{NP}$ where NP is the number of penetrators. The number of kills is also binomial, with parameters n, the number of attacks; and p, the probability of kill. There will be i kills out of j detections where $j \geq i$. The number of attacks made with j detections is essentially the classical occupancy problem considered in Appendix D.

Of j detections, there can be just one unique detection where all interceptors detect the same penetrator; or each interceptor can see a different penetrator. If the number of interceptors

exceeds the number of penetrators, there are some duplicate detections. So, of j detections there can be up to the minimum of (NP, NAI) attacks made, where NP is the total number of penetrators, and NAI is the total number of AIs.

The general form of the distribution of X, the number of kills, is:

$$P(X=i) = \sum_{j=i}^{NAI} [\binom{NAI}{j} [1-(1-P_d)^{NP}]^j [(1-P_d)^{NAI-j} \left\{ \sum_{s} \binom{s}{i} P_k^i (1-P_k)^{s-i} P_{NP-s}(j,NP) \right\}] \quad (C.11)$$

where

NP is the total number of penetrators,

NAI is the total number of interceptors,

i is the number of kills i=[0,1,...,min(NP,NAI)

j is the number of detections j=(i,i+1,...,NAI)

P_d is the probability of detecting a single penetrator,

s is the number of unique detections

$$\text{lower limit} = \begin{cases} 1 & \text{if } i=0,1; \text{ upper limit} = \min(j,NP) \\ i & \text{o.w.} \end{cases}$$

P_k is the probability of kill,

$$P_{NP-s}(j,NP) = \binom{NP}{NP-s} \sum_{v=0}^{s} (-1)^v \binom{s}{v} \left(1 - \frac{(NP-s)+v}{NP}\right)^j \quad (C.12)$$

m = (NP-s) empty cells; penetrators without an AI assigned,

r = NAI interceptors, and

n = NP penetrators.

In using this distribution, integer penetrators and

interceptors are used. However, at each time interval the expected number of penetrators and interceptors is used and may not be integer valued. Thus to apply the distribution, this must be solved for the upper and lower integer values of number of penetrators and AIs and then appropriately weighted. For example, suppose there are 3.5 penetrators and 2.2 interceptors then the distribution is solved for four cases:

	penetrators	interceptor
case 1	3	2
case 2	3	3
case 3	4	2
case 4	4	3

The weight applied to solution of each case is developed as shown below:

	P(2 AIs)=.8	P(3 AIs)=.2
P(3 pen)=.5	.4	.1
P(4 pen)=.5	.4	.1

APPENDIX D

THE CLASSICAL CELL OCCUPANCY PROBLEM

This appendix provides the development of the cell occupancy problem as presented by Feller in Ref. 9, which is used in the development of the autonomous model. The classical cell occupancy problem places r balls in n cells assuming each arrangement occurs with equal probability, n^{-r}, and then finds the probability that m cells are empty.

First define A_i as the event that cell i is empty, (i = 1, 2,...,n). Then the r balls are placed in any of the remaining n-1 cells, regardless of how many other balls a cell contains. The first ball may be placed in any of the remaining n-1 cells, the second ball also may be placed in any of the remaining n-1 cells. Since there are r balls, there are $(n-1)^r$ different arrangements which leave the i^{th} cell empty. Similarly, for an event A_{ij} where two cells are empty (i = 1,2,...,n and j = 1,2,...,n, j=i), the r balls are distributed among the remaining n-2 cells. So there are $(n-2)^r$ arrangements possible which leave the two cell i, j, empty.

The probability a ball is placed in any cell i is (1/n), and the probability a ball is not placed in cell i is (1 - (1/n)). For the cell to remain empty, none of the r balls must be placed in this cell, so the probability i is empty is $(1 - (1/n))^r$. The same argument can be made for the probability of 2,3,...,n cells remaining empty.

$$P_i = (1 - (1/n))^r \tag{D.1}$$

$$P_{ij} = (1 - (2/n))^r \tag{D.2}$$

$$P_{ijk} = (1 = (3/n))^r \tag{D.3}$$

Finally, consider not only the probability of cell i being empty, but the number of ways just one cell could be empty. Since i could be anyone of n cells, ($i = 1,2,\ldots,n$), there are $\binom{n}{1}$ different ways to leave one cell empty. If two cells are empty, then there are $\binom{n}{2}$ combinations which leave two cells empty. Hence for every v n, define:

$$S_v = \binom{n}{v} (1 - v/n))^r \tag{D.4}$$

The probability all cells are occupied is one minus the sum of the probabilities that 1,2,3,...,n cells are empty. However, in this problem the events overlap; the event of two empty cells also includes the event of one empty cell. Returning to some simple rules of set theory to find the probability of a sum of overlapping events, let A_1 and A_2 denote two events and let $A = A_1 \cup A_2$. Then,

$$P(A) = P(A_1) + P(A_2) - P(A_1A_2) \tag{D.5}$$

A generalization of this can be made to N events. Define,

$$A = A_1 \cup A_2 \cup \ldots \cup A_N \tag{D.6}$$

then the probability of one event, two events, etc., is:

$$P_i = P(A_i) \quad P_{ij} = P(A_iA_j) \quad P_{ijk} = P(A_iA_jA_k) \tag{D.7}$$

and summing over all subscripts:

$$S_1 = \sum P_i \quad S_2 = \sum P_{ij} \quad S_3 = \sum P_{ijk} \quad i \; j \; k \ldots N \tag{D.8}$$

Then the probability of the realization of at least one among the events $A_1A_2,\ldots,A_N$ is given by:

$$P_1 = S_1 - S_2 + S_3 -+ \ldots S_N \tag{D.9}$$

This result can be applied to find the probability that all n cells are occupied by r balls:

$$
\begin{aligned}
P_0(r,n) &= 1 - P_1 \\
&= 1 - S_1 + S_2 - S_3 +- \ldots . \; S_N \\
&= \sum (-1)^v \binom{n}{v} (1 - (v/n))^r \qquad \text{(D.10)}
\end{aligned}
$$

where $P_0(r,n)$ represents the probability that 0 cells are empty when r balls are p'.ced in n cells.

Consider a distribution where exactly m cells are empty. The r balls will be distributed among the (n-m) remaining cells, and there are $\binom{n}{m}$ ways to choose the m empty cells. The number of such distributions is $(n-m)^r P_0(r,n-m)$. Dividing by n^r, the probability that exactly m cells remain empty is:

$$
\begin{aligned}
P_m(r,n) &= \binom{n}{m} (1 - (m/n))^r P_0(r,n-m) \\
&= \binom{n}{m} \sum (-1)^v \binom{n-m}{v} (1 - ((m+v)/n))^r \qquad \text{(D.11)}
\end{aligned}
$$

In this study the probability of I different detections which result from the assignment of r interceptors to n penetrators can be solved using the result of the classical occupancy problem. The interceptors represent the r balls, and the penetrators are the n cells. The number of empty cells, m, is the number of penetrators which do not have interceptors assigned. Conversely, (n-m) cells is the number of penetrators which are assigned interceptors. So this distribution can be solved for s = (n-m) to find the number of unique penetrators which have interceptors assigned, and hence the number of attacks which can be made.

VITA

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18. SUPPLEMENTARY NOTES
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19. KEY WORDS (*Continue on reverse side if necessary and identify by block number*)
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This study develops a basic methodology for modeling the effects of command and control on the Forward Air Defense (FAD). It is modeled from the Soviet perspective to judge the effectiveness of the defense against a US penetrating force.

Three possible Soviet defenses which might occur in the FAD are postulated: a SUAWACS, a super AI or autonomous operation. The first represents a best case defense where interceptors operate under the close control of the Soviet Union Airborne Warning and Control System (SUAWACS). The worst case postulated is an autonomous interceptor force which must depend on their own limited capabilities to detect penetrators. The Soviet Union has a large number of interceptors with a variety of capabilities. An intermediate position which would provide some degree of command and control is a super AI. A super AI is an interceptor with advanced capabilities that could be used to control a small number of interceptors.

A analytic, expected value methodology is developed and then adapted to the various command and control defenses described. The measure of effectiveness used is the expected number of survivors. The model provides a number of inputs which can be changed to determine parameter sensitivity. The model is based on a grid structure to develop the geometry involved in intercepts, similar to that used in the COPEM model. This is one of the first models to explicitly model a parameter for control channels. Another parameter used to capture differences in command and control is the C^2 cycle time.